EE 435

Lecture 41

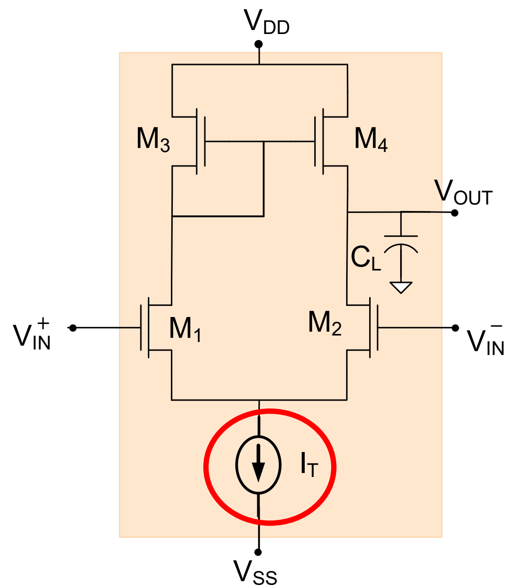
References and Bias Generators

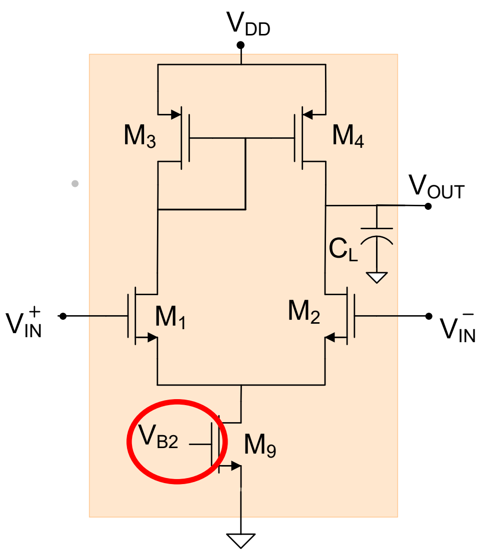
Bias Voltages/Currents and References

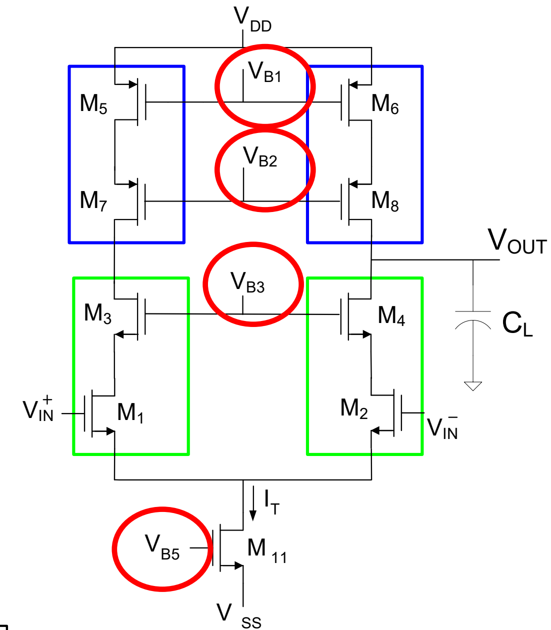
How do we get quantities such as voltage, current, resistance, temperature, ?.... in an electronic circuit

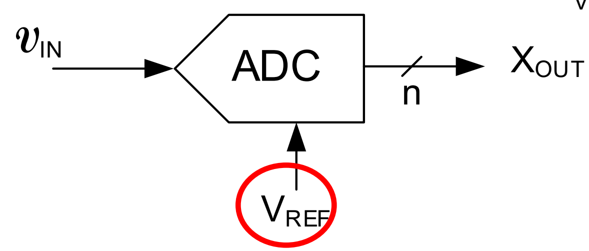
Bias Voltages/Currents and References

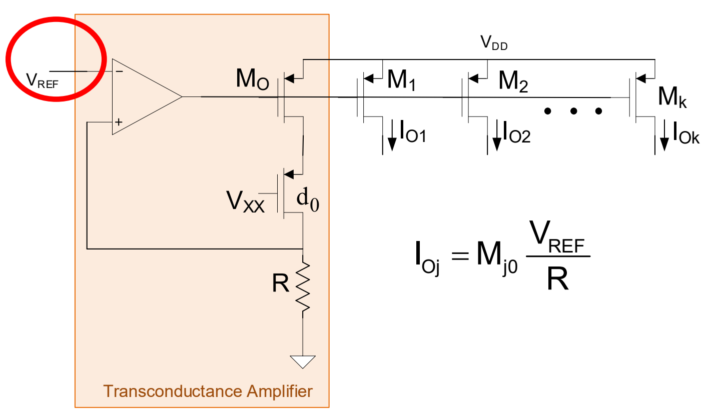
How are these voltages and currents generated?



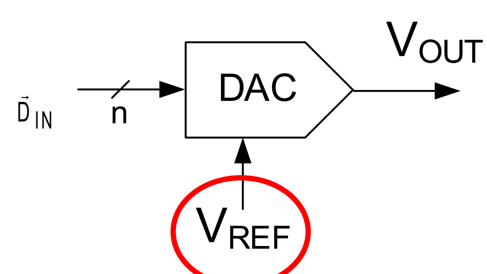






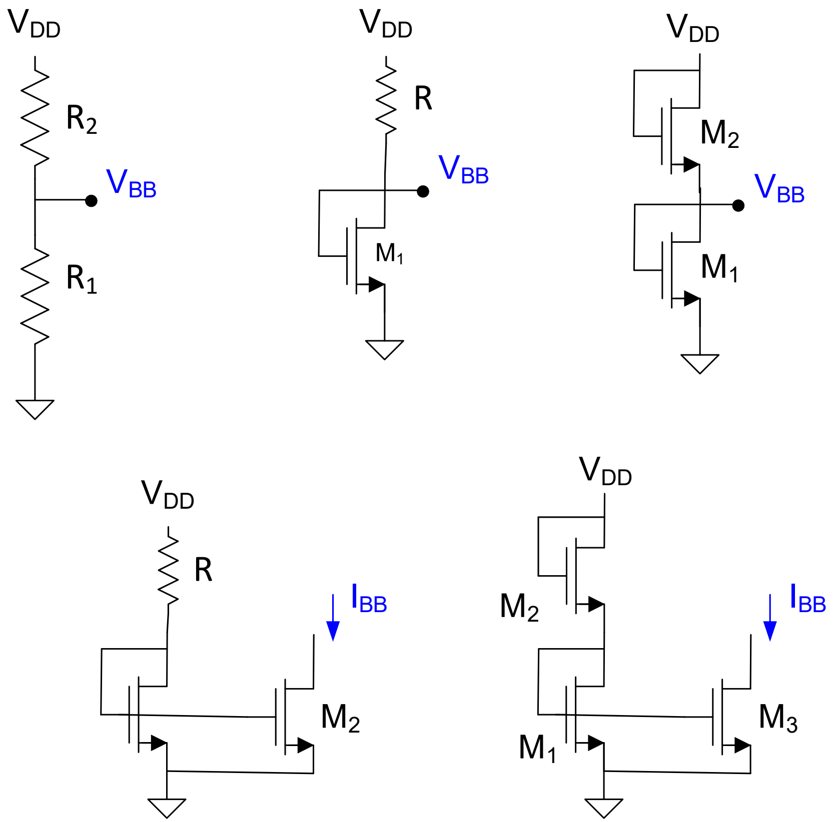


$$I_{Oj} = M_{j0} \frac{V_{REF}}{R}$$



Bias Voltages/Currents and References

How are these voltages and currents generated?



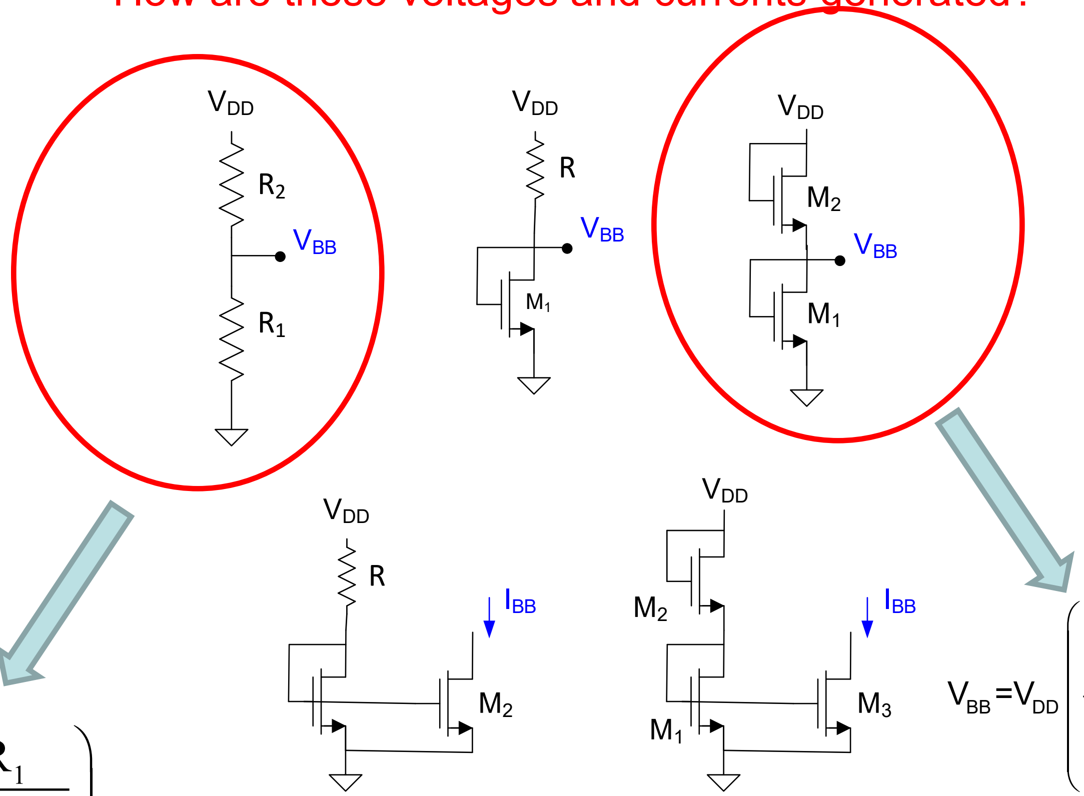
All will work !

Termed Supply-Referenced Sources

But supply sensitivity (supplies usually poorly controlled and noisy), process dependence, and temperature dependence unacceptable in many applications

Bias Voltages/Currents and References

How are these voltages and currents generated?



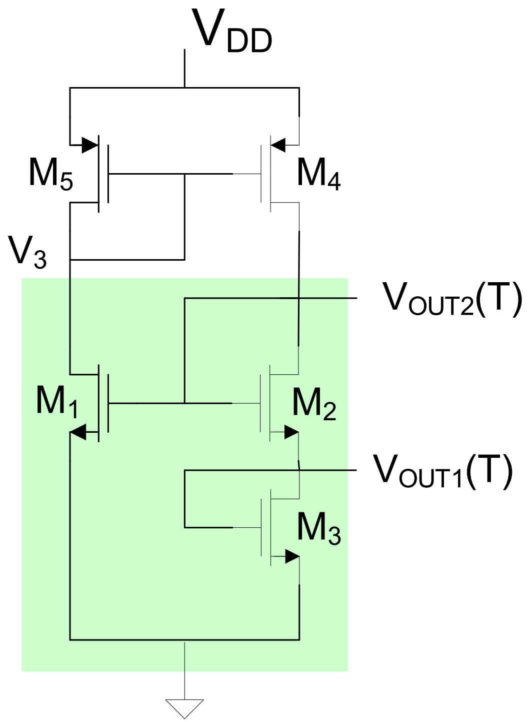
$$V_{BB} = V_{DD}\left(\frac{R_1}{R_1 + R_2}\right)$$

$$V_{BB} = V_{DD}\left(\frac{\sqrt{\frac{W_2 L_1}{W_1 L_2}}}{1+\sqrt{\frac{W_2 L_1}{W_1 L_2}}}\right) + V_{TH}\left(\frac{1-\sqrt{\frac{W_2 L_1}{W_1 L_2}}}{1+\sqrt{\frac{W_2 L_1}{W_1 L_2}}}\right)$$

For voltage references, must find circuit that generates output that has units Volts !
For current references, must find circuit that generates output that has units Amps !

Bias Voltages/Currents Generators

How are these voltages and currents generated?



Inverse-Widlar

$$V_{01} = V_{Tn}\left(\frac{1-\sqrt{\dfrac{M_{54}W_2L_1}{W_1L_2}}}{1+\sqrt{\dfrac{W_2L_3}{W_3L_2}}-\sqrt{\dfrac{M_{54}W_2L_1}{W_1L_2}}}\right)$$

$$V_{02} = V_{Tn}\left(\frac{1+\sqrt{\dfrac{W_2L_3}{W_3L_2}}-2\sqrt{\dfrac{M_{54}W_2L_1}{W_1L_2}}}{1+\sqrt{\dfrac{W_2L_3}{W_3L_2}}-\sqrt{\dfrac{M_{54}W_2L_1}{W_1L_2}}}\right)$$

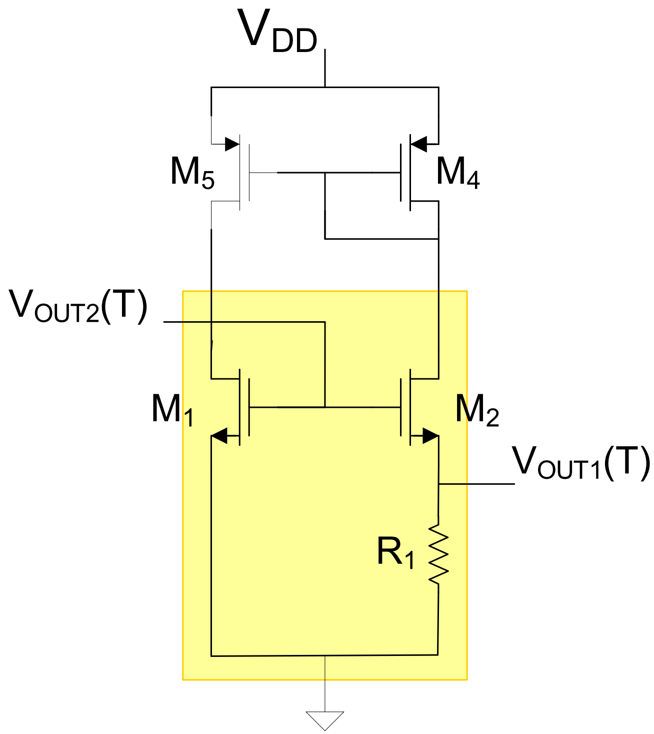
M_{54} is the M_5:M_4 Current Mirror Gain

Supply-independent Bias Generator!

Start-up circuit needed (notice positive feedback loop)

Supply-independent Bias Generators Widely Used

Bias Voltages/Currents Generators



$$V_{01} = \left(\frac{\theta_1}{2} \pm \sqrt{\frac{\theta_1 V_{Tn}}{2} + \left(\frac{\theta_1}{2}\right)^2} \right) \left(1 - \sqrt{\frac{W_1 L_2}{M_{45} W_2 L_1}} \right)$$

$$V_{02} = V_{Tn} + \frac{\theta_1}{2} \pm \sqrt{\frac{\theta_1 V_{Tn}}{2} + \left(\frac{\theta_1}{2}\right)^2}$$

where

$$\theta_1 = \frac{M_{45} 2 L_1}{R \mu_n C_{OX} W_1}$$

Widlar Generator !

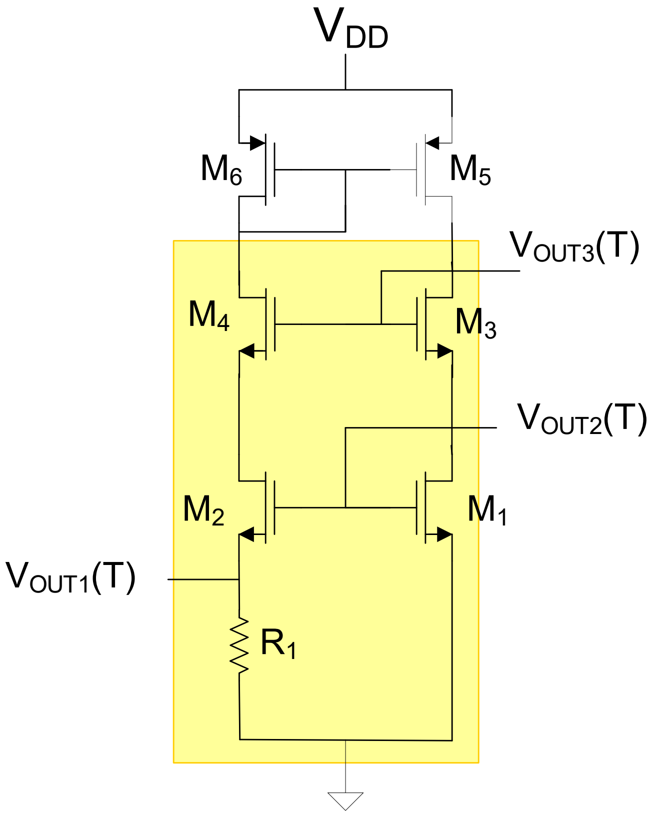
M_{45} is the M_4:M_5 Current Mirror Gain

Supply-independent Bias Generator!

Start-up circuit needed (notice positive feedback loop)

Supply-independent Bias Generators Widely Used

Bias Voltages/Currents Generators



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$$V_{01} = \left(\frac{\theta_1}{2} \pm \sqrt{\frac{\theta_1 V_{Tn}}{2} + \left(\frac{\theta_1}{2}\right)^2}\right)\left(1 - \sqrt{\frac{W_1 L_2}{M_{65} W_2 L_1}}\right)$$

$$V_{02} = V_{Tn} + \frac{\theta_1}{2} \pm \sqrt{\frac{\theta_1 V_{Tn}}{2} + \left(\frac{\theta_1}{2}\right)^2}$$

where

$$\theta_1 = \frac{M_{65} 2 L_1}{R \mu_n C_{OX} W_1}$$

M_{65} is the M_6:M_5 Current Mirror Gain

Widlar Generator !

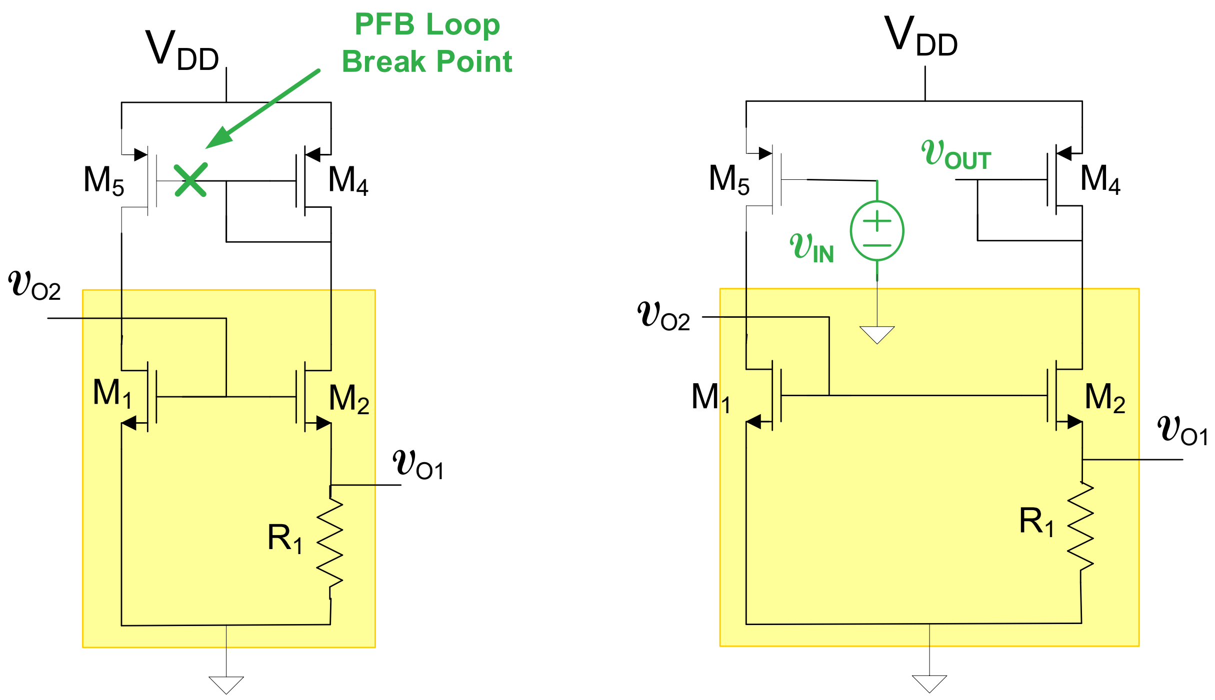
Supply-independent Bias Generator!

Start-up circuit needed (notice positive feedback loop)

Supply-independent Bias Generators Widely Used

Bias Voltages/Currents Generators

Need for Start-up Circuit



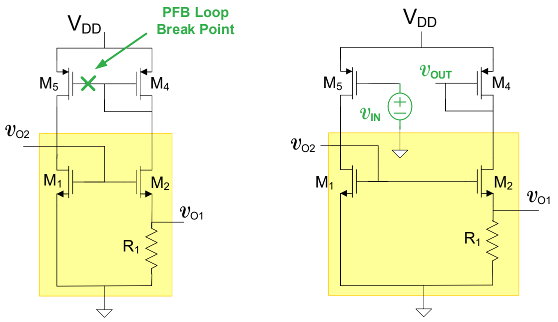
V_{OUT}=f(V_{IN}) termed the return map

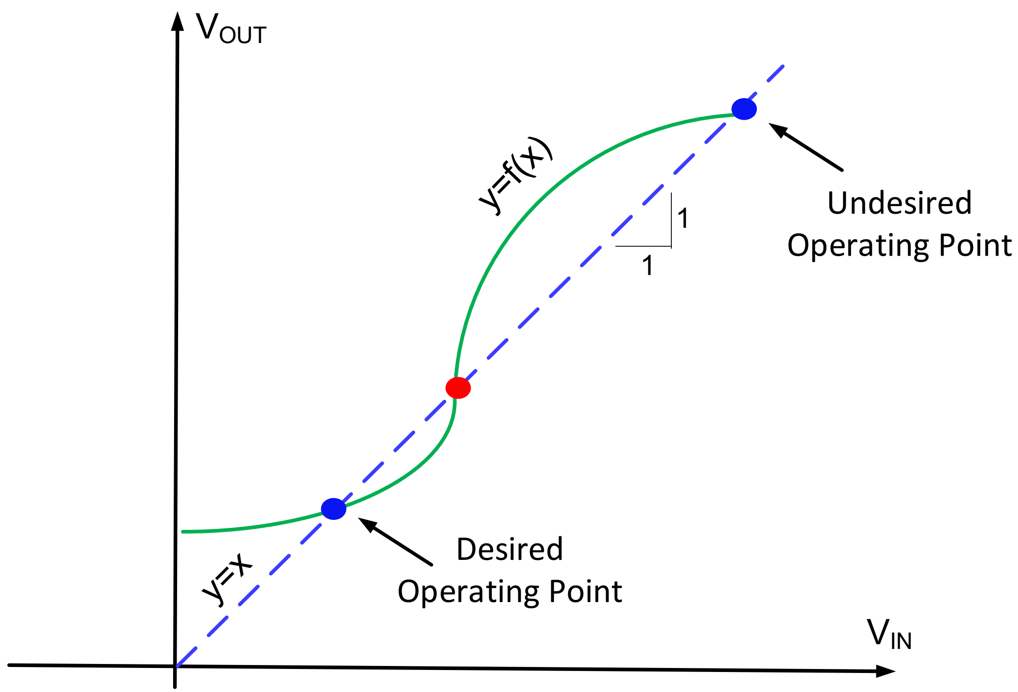
Termed Homotopy Analysis

Must not perturb operating point when breaking loop !

Bias Voltages/Currents Generators

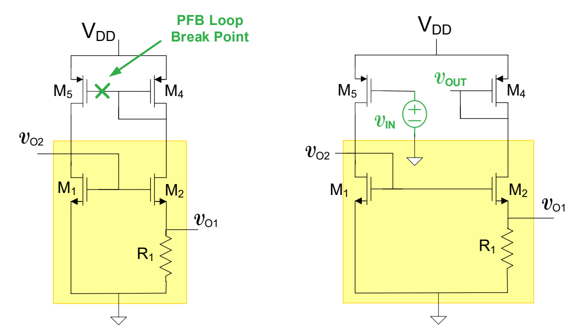
Need for Start-up Circuit

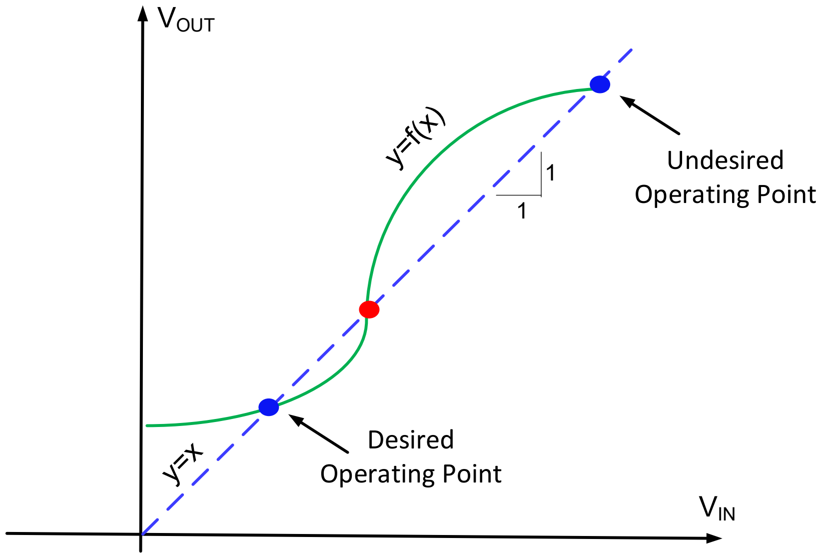




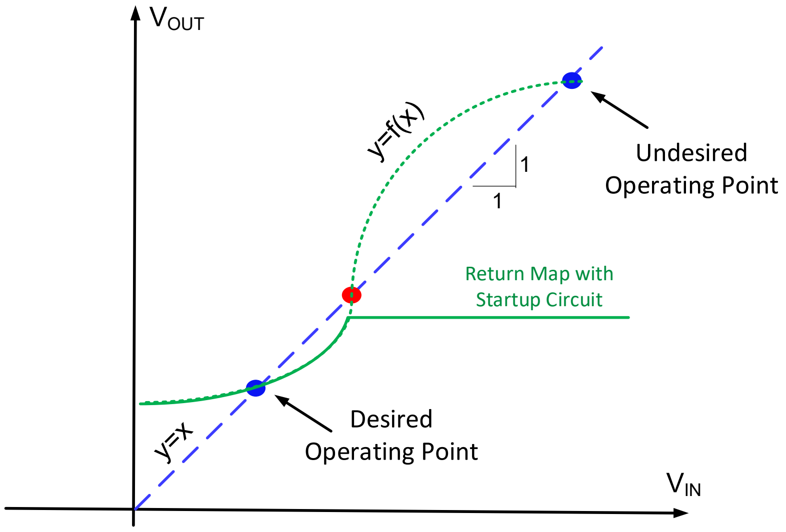
Bias Voltages/Currents Generators

Need for Start-up Circuit





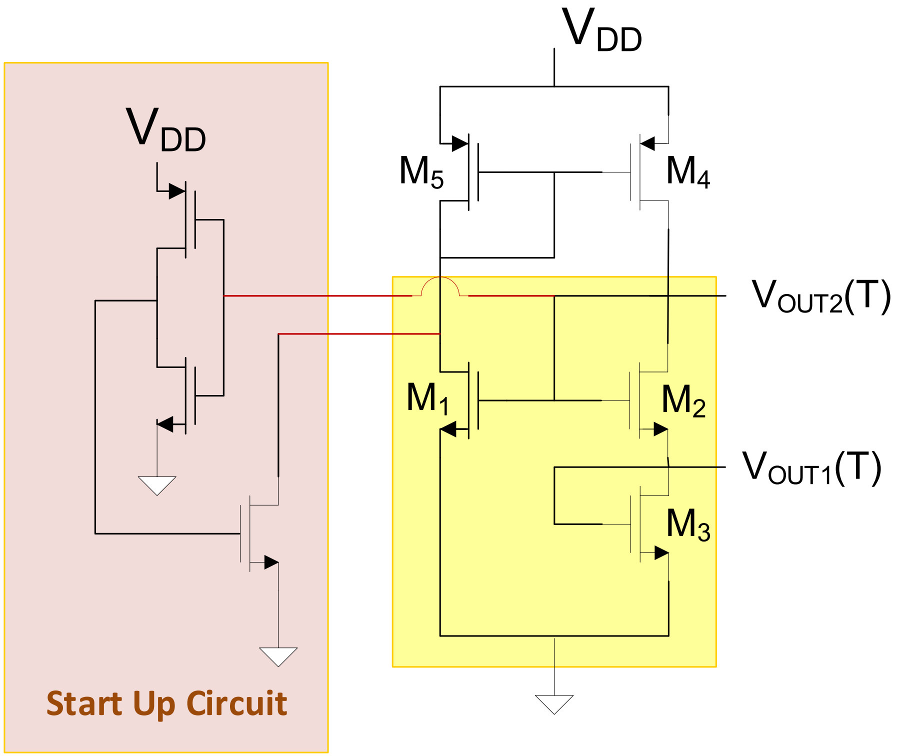
Without start-up circuit



With start-up circuit

Must verify start-up is effective over PVT variations

Bias Voltages/Currents Generators



Several different start-up circuits have been used

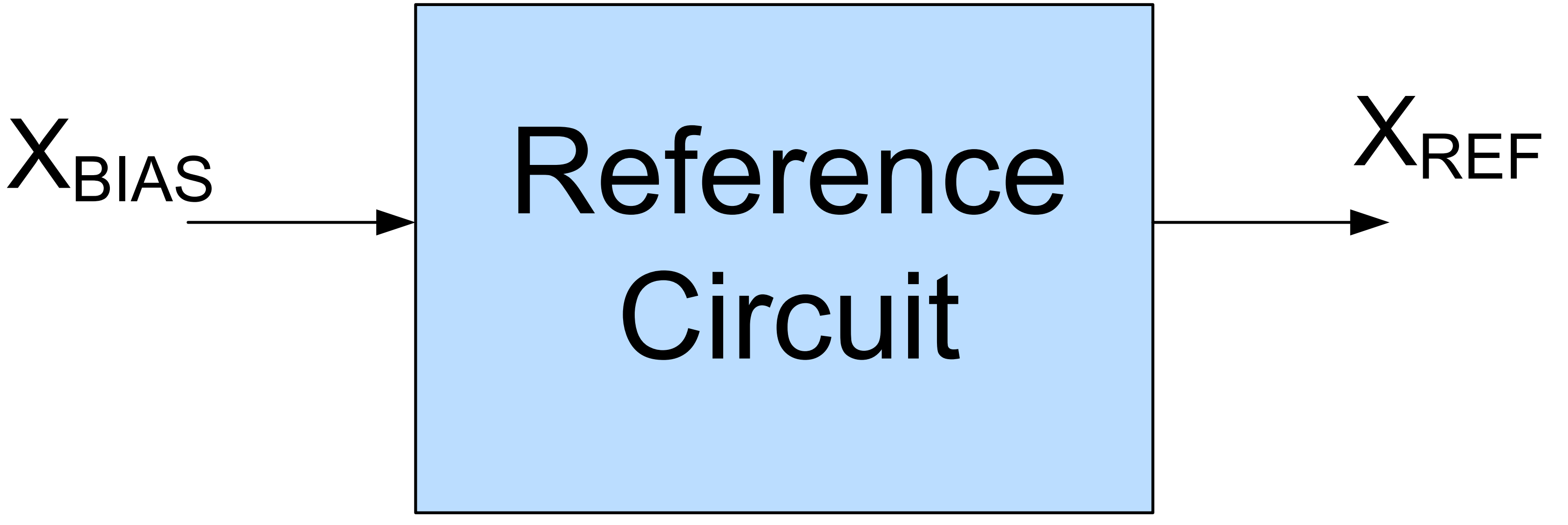
This start-up circuit shuts off during normal operation !

Types of References

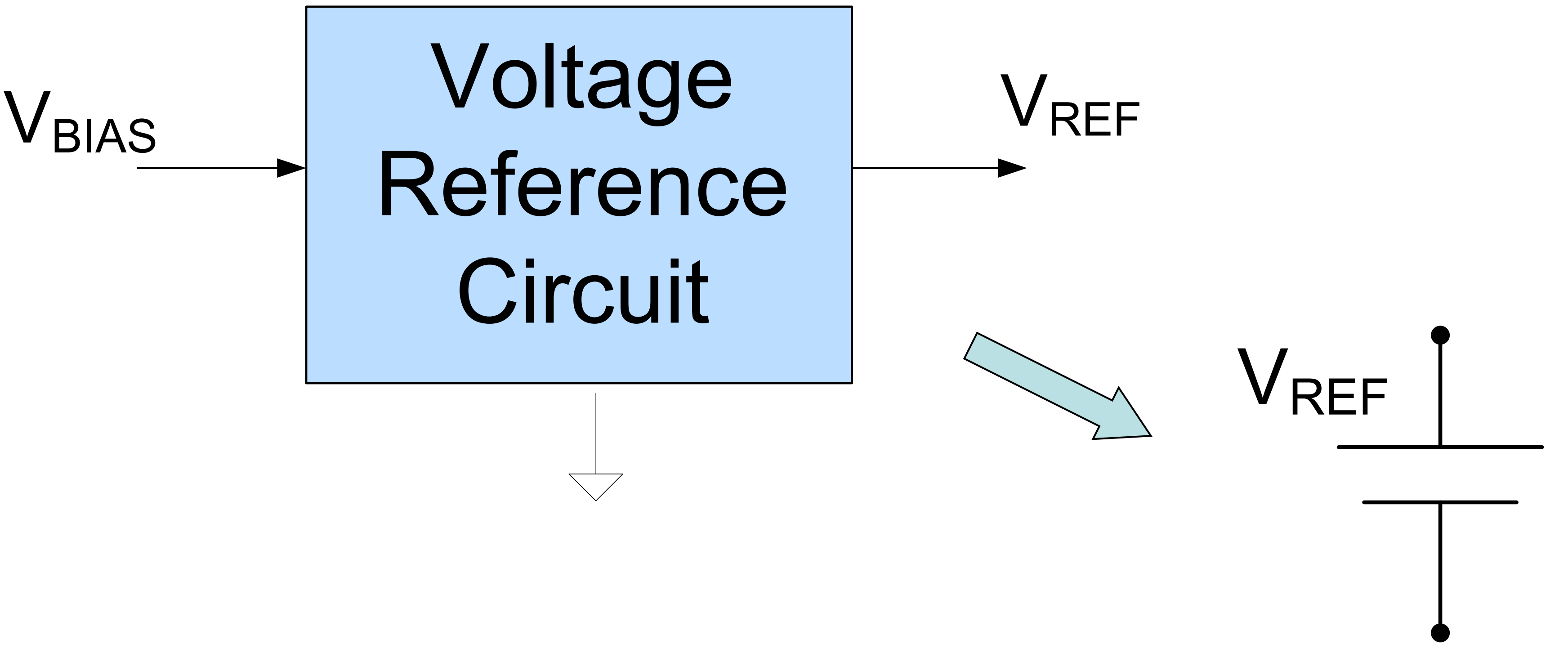
- Voltage References
- Current References
- Time References
- ….

Sensors Closely Related

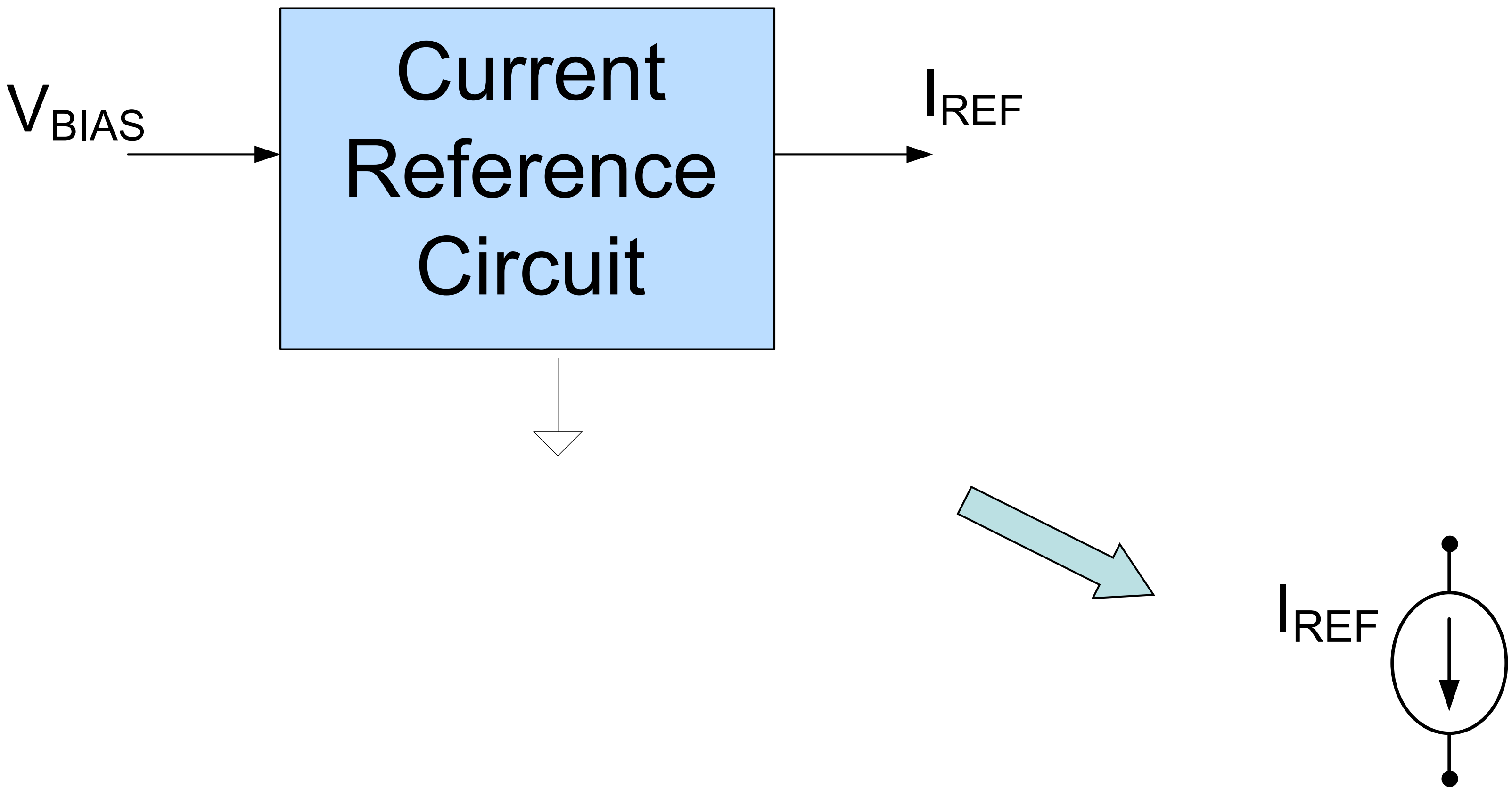
- Temperature
- Period
- Resistance
- Capacitance
- ….


X_{BIAS}
Reference
Circuit
X_{REF}

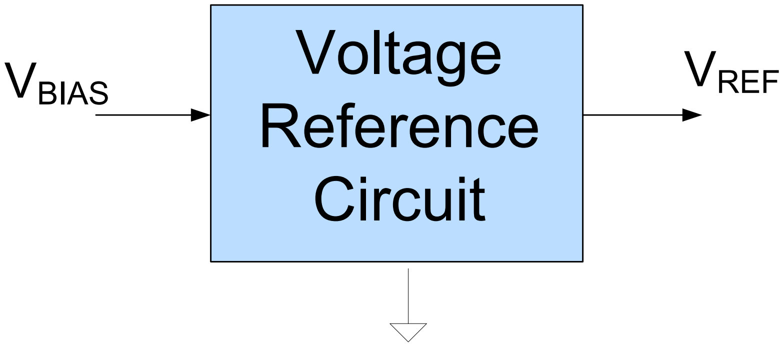
Voltage Reference



Current Reference



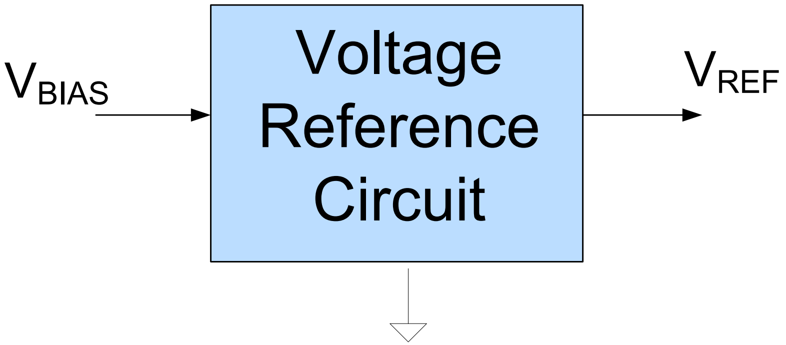
Desired Properties of References



- Accurate
- Temperature Stable
- Time Stable
- Insensitive to V_{BIAS}

- Low Output Impedance (voltage reference)
- Floating
- Small Area
- Low Power Dissipation
- Process Tolerant
- Process Transportable

Desired Properties of References

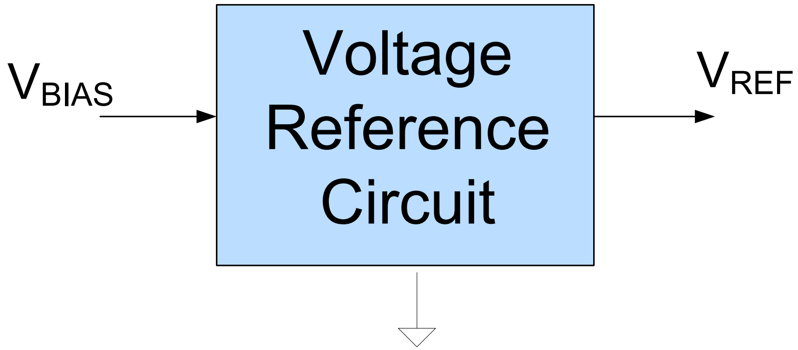


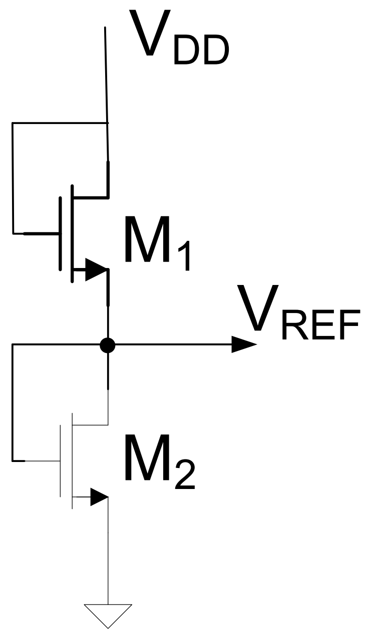
- Accurate √
- Temperature Stable √
- Time Stable √
- Insensitive to V_{BIAS} √

- Low Output Impedance (voltage reference)
- Floating
- Small Area
- Low Power Dissipation
- Process Tolerant √
- Process Transportable

Similar properties desired in other references

Consider Voltage References





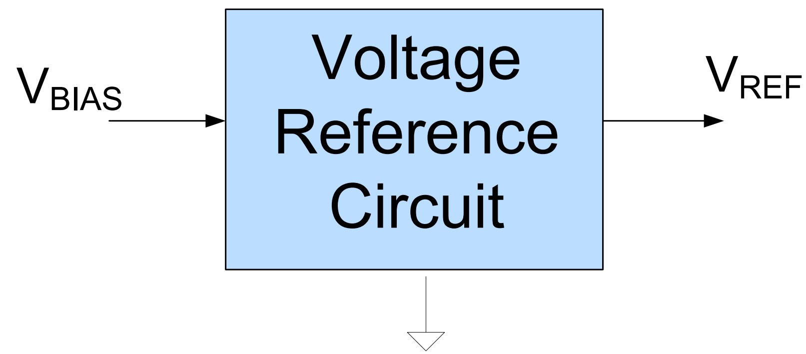
Popular Voltage “Reference”

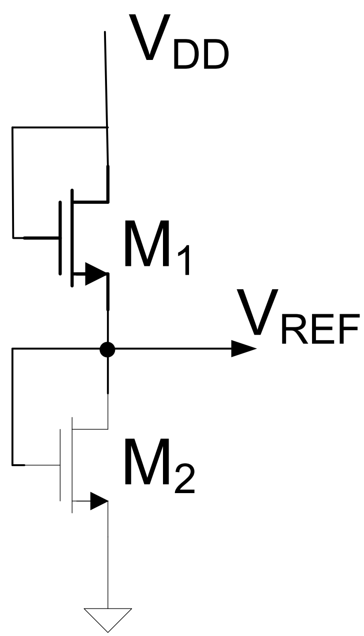
$$\left.\begin{aligned}
I_{D1} &= \frac{\mu C_{OX} W_1}{2L_1}\left(V_{GS1} - V_{T1}\right)^2 \\
I_{D2} &= \frac{\mu C_{OX} W_2}{2L_2}\left(V_{GS2} - V_{T2}\right)^2 \\
V_{T1} &= V_{TH0} + \gamma\left(\sqrt{\varphi + V_{REF}} - \sqrt{\varphi}\right) \\
V_{DD} - V_{REF} - V_{T1} &= \sqrt{\frac{W_2 L_1}{W_1 L_2}}\left(V_{REF} - V_{T2}\right)
\end{aligned}\right\}$$

If matching assumed and γ effects neglected

$$V_{REF} = \frac{V_{DD} - V_{TH0}\left(1 - \sqrt{\frac{W_2 L_1}{W_1 L_2}}\right)}{1 + \sqrt{\frac{W_2 L_1}{W_1 L_2}}}$$

Consider Voltage References





Popular Voltage "Reference"

Uses as a reference limited to biasing and even for this may not be good enough !

If matching assumed and γ effects neglected

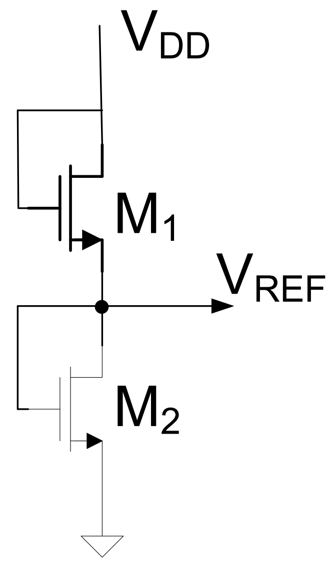
$$V_{REF} = \frac{V_{DD} - V_{TH0}\left(1 - \sqrt{\frac{W_2 L_1}{W_1 L_2}}\right)}{1 + \sqrt{\frac{W_2 L_1}{W_1 L_2}}}$$

Dependent upon V_{DD}, V_{TH0}, matching, process variations, γ

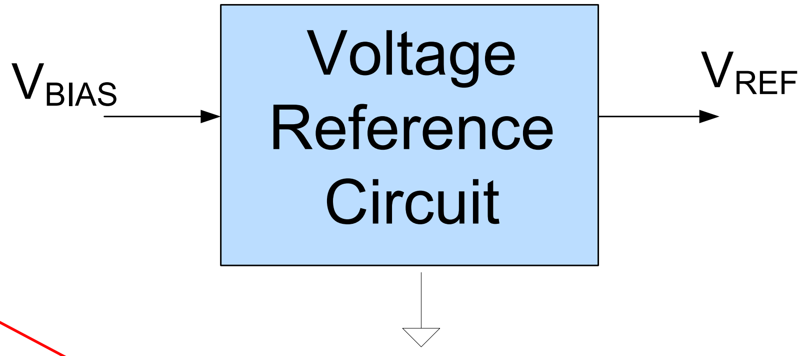
Termed a V_{DD},V_{TH} reference

Does not satisfy key properties of voltage references

Consider Voltage References



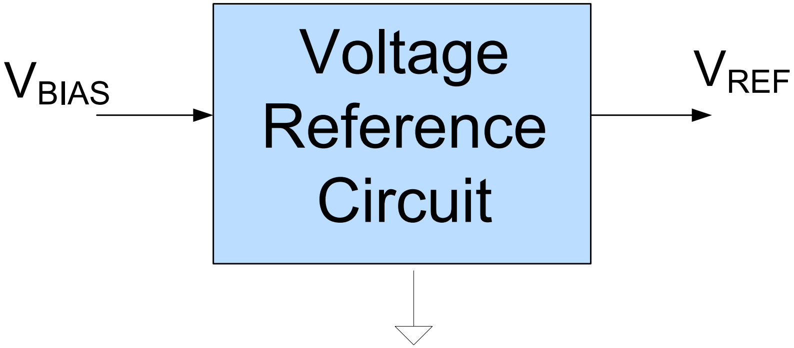
V_{DD},V_T reference



$$V_{REF} = \frac{V_{DD} - V_{T0}\left(1 - \sqrt{\frac{W_2 L_1}{W_1 L_2}}\right)}{1 + \sqrt{\frac{W_2 L_1}{W_1 L_2}}}$$

Observation – Variables with units Volts needed to build any voltage reference

Voltage References



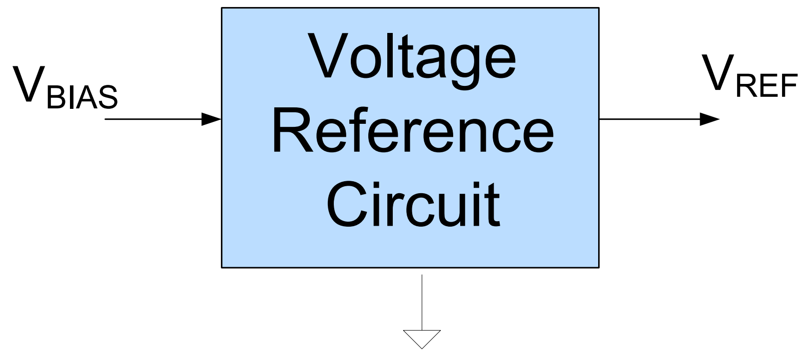
Observation – Variables with units Volts needed to build any voltage reference

What variables available in a process have units volts?

What variables which have units volts satisfy the desired properties of a voltage reference?

How can a circuit be designed that "expresses" the desired variables?

Voltage References



Observation – Variables with units Volts needed to build any voltage reference

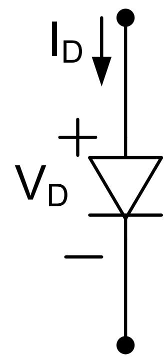
What variables available in a process have units volts?

V_{DD}, V_T, V_D (diode) ,V_Z,V_{BE},V_t ,V_{TH} ???

What variables which have units volts satisfy the desired properties of a voltage reference?

How can a circuit be designed that "expresses" the desired variables?

Voltage References



Consider the Diode

$$I_D = J_S A e^{\frac{V_D}{V_t}}$$

$$J_S = \tilde{J}_{SX}\left[T^m e^{\frac{-V_{G0}}{V_t}}\right]$$

$$V_t = \frac{kT}{q}$$

$$\frac{k}{q} = \frac{1.38x10^{-23}}{1.602x10^{-19}}\frac{V}{^oK} = 8.614x10^{-5}\frac{V}{^oK}$$

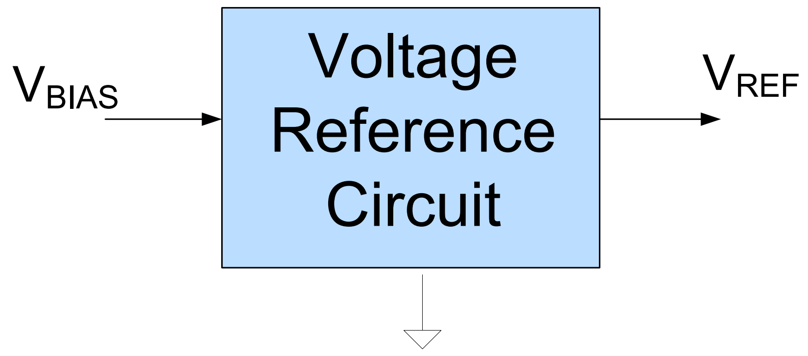
$$V_{G0} = 1.206V$$

termed the bandgap voltage

pn junction characteristics highly temperature dependent through both the exponent and J_S

V_{G0} is nearly independent of process and temperature

Voltage References



Observation – Variables with units Volts needed to build any voltage reference

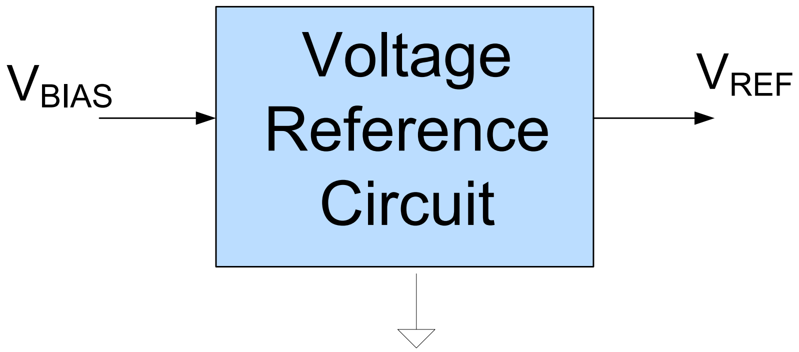
What variables available in a process have units volts?

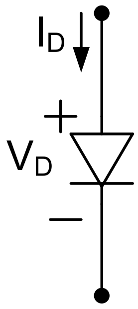
V_{DD}, V_T, V_D (diode) ,V_Z,V_{BE},V_t ,V_{G0} ???

What variables which have units volts satisfy the desired properties of a voltage reference? V_{G0} and ??

How can a circuit be designed that “expresses” the desired variables?

Voltage References





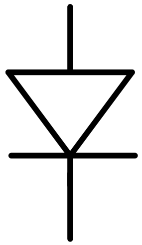
V_{DD}, V_T, V_D (diode) ,V_Z,V_{BE},V_t ,V_{G0} ???

How can a circuit be designed that "expresses" the desired variables?

- V_{G0} is deeply embedded in a device model with horrible temperature effects !
- Good diodes are not widely available in most MOS processes !

$$I_C = \tilde{J}_{SX} A T^m e^{\frac{-V_{G0}}{V_t}} e^{\frac{V_{BE}}{V_t}}$$

Voltage References



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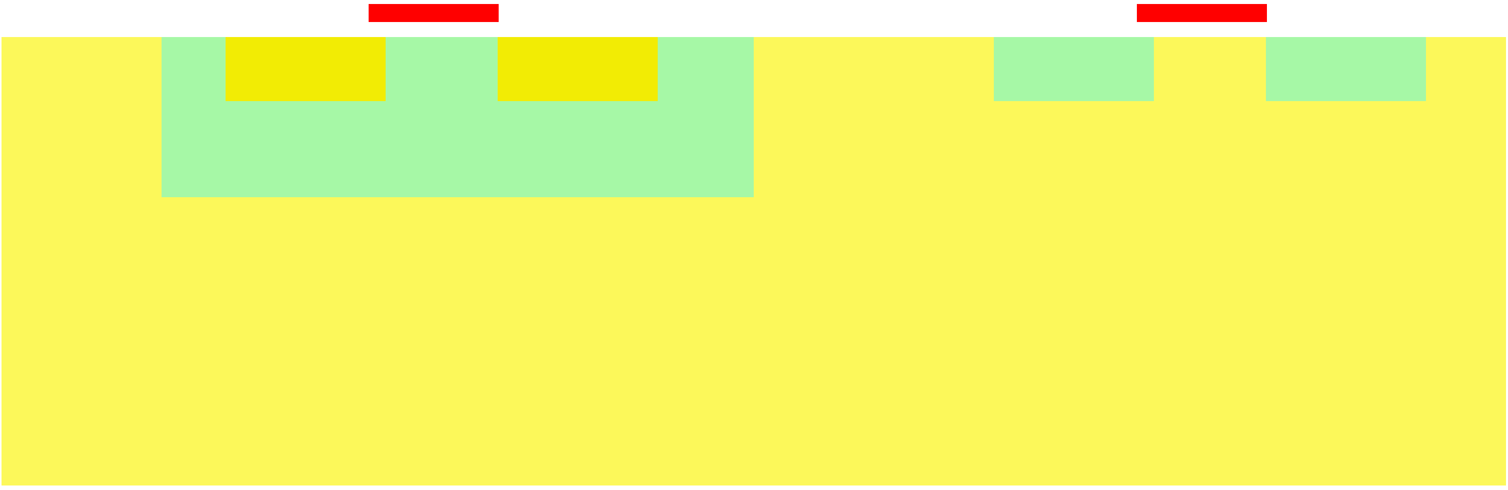
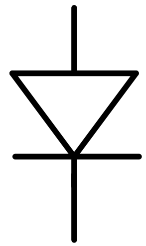


Voltage References



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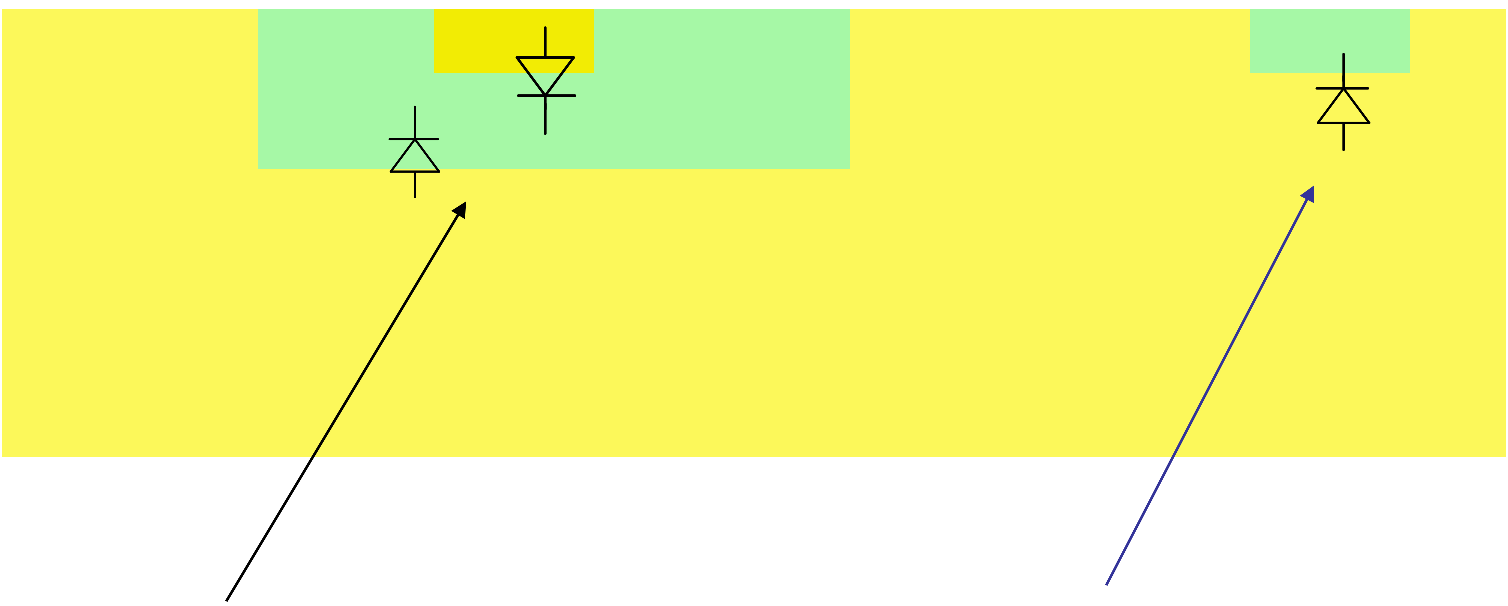
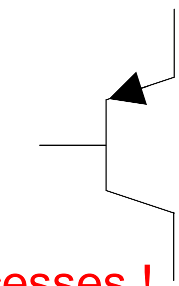


These diodes interact and
actually form substrate pnp transistor

Not practical to forward bias junction

Voltage References



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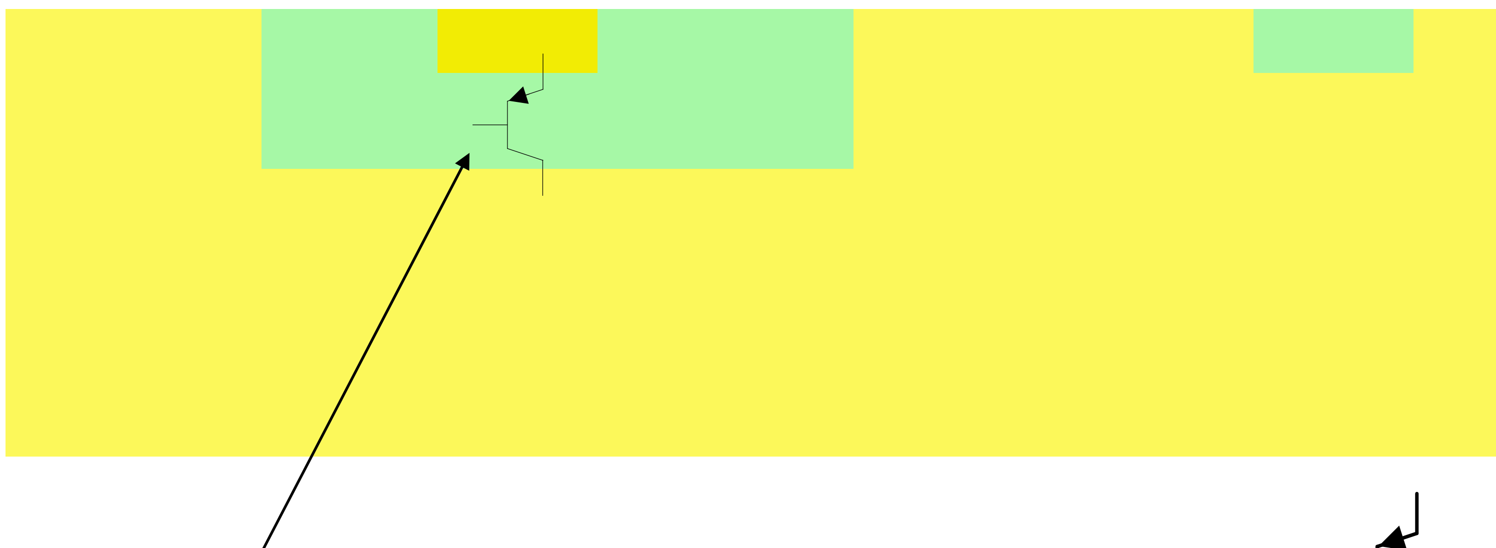
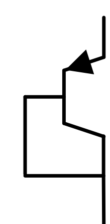
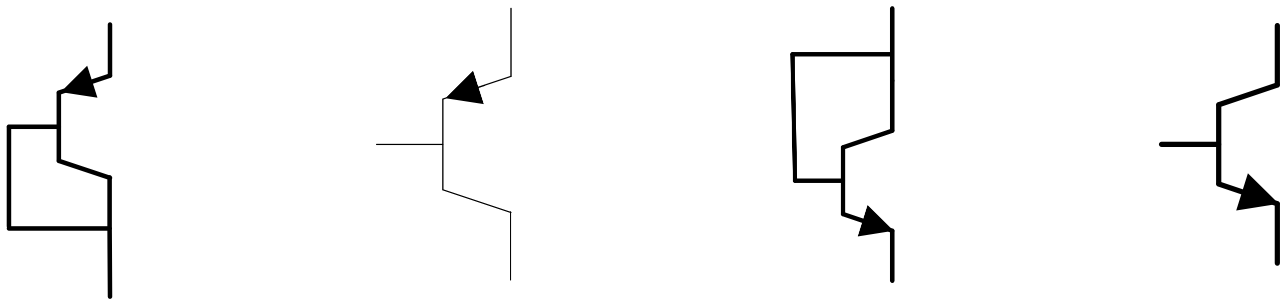


Substrate pnp transistor

Diode-connected substrate pnp



Voltage References



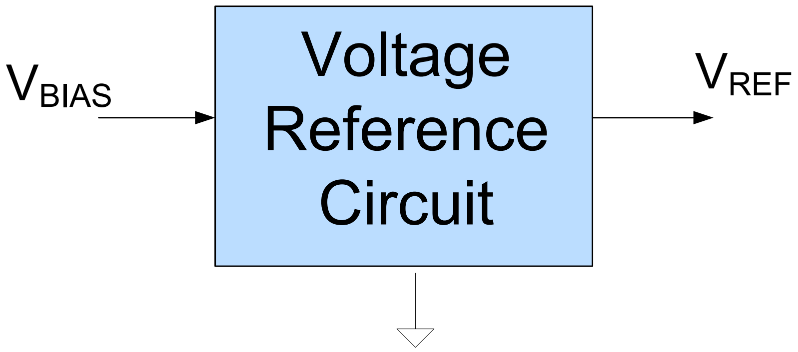
$$I_C = J_S A e^{\frac{V_{BE}}{V_t}}$$

$$J_S = \tilde{J}_{SX}\left[T^m e^{\frac{-V_{G0}}{V_t}}\right]$$

Bandgap Voltage Appears in BJT Model Equation as well

$$I_C(T) = \left(\tilde{J}_{SX} A\left[T^m e^{\frac{-V_{G0}}{V_t}}\right]\right) e^{\frac{V_{BE}(T)}{V_t}}$$

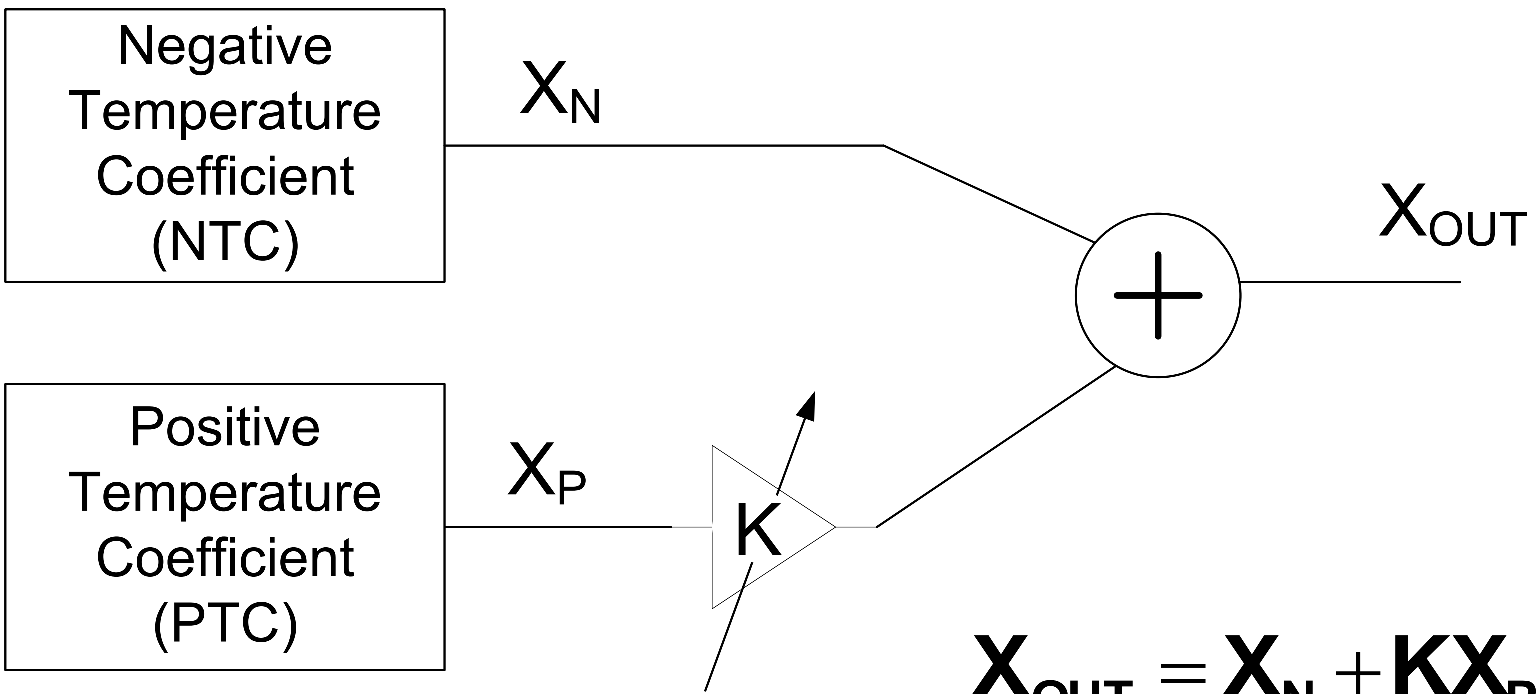
Voltage References



Voltage references that "express" the bandgap voltage are termed "Bandgap References"

- V_{G0} is deeply embedded in a device model with horrible temperature effects !
- Good BJTs are not widely available in most MOS processes but the substrate pnp is available !

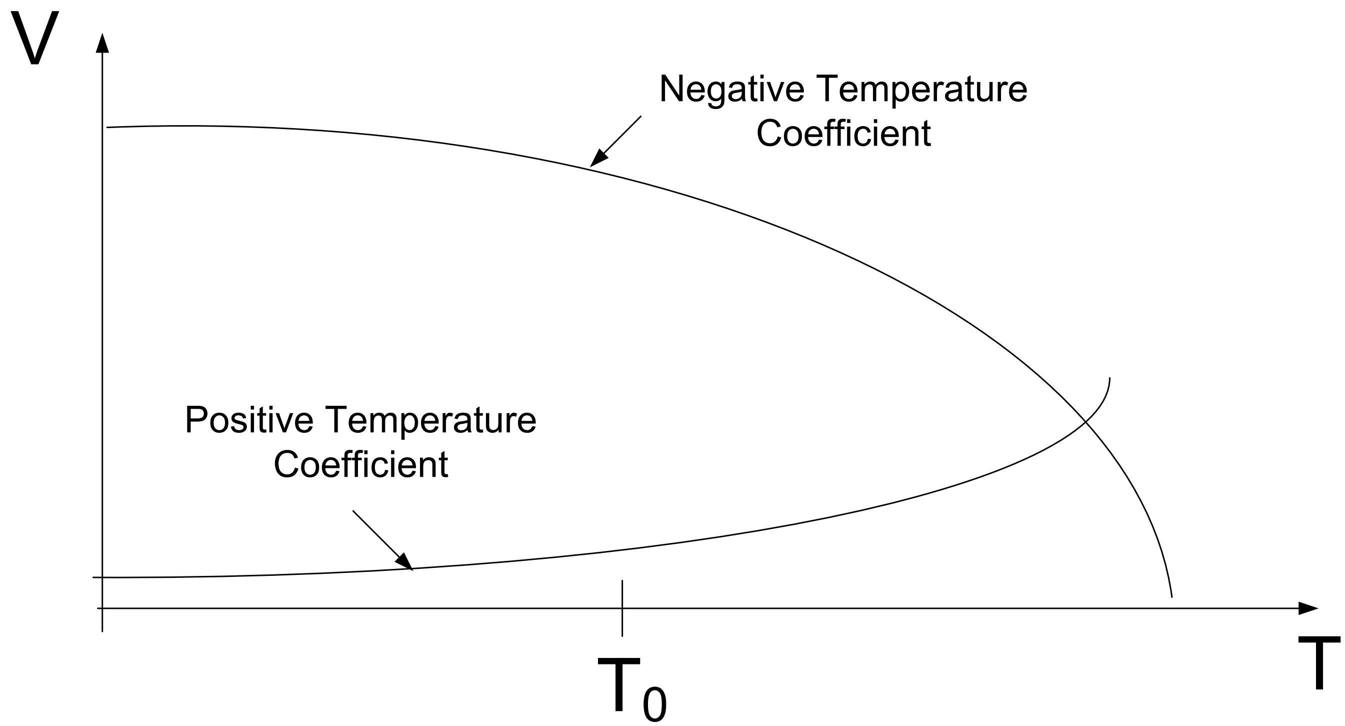
Standard Approach to Building Voltage References



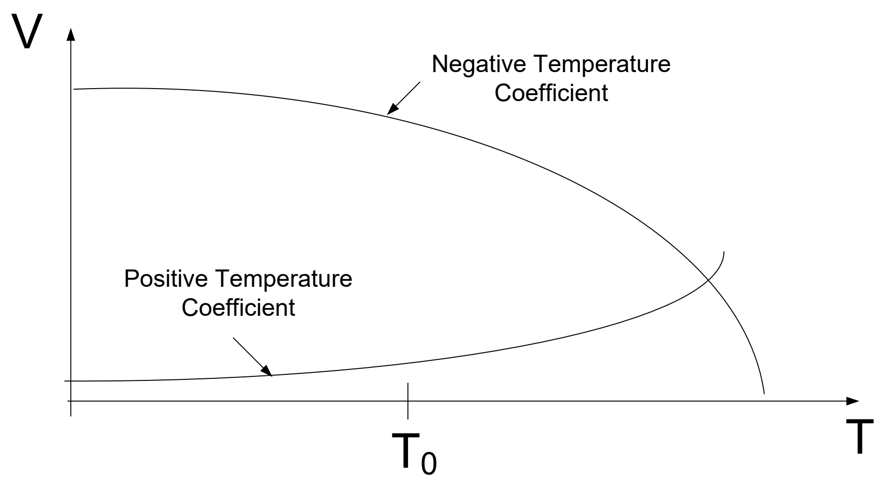
$$\mathbf{X_{OUT} = X_N + KX_P}$$

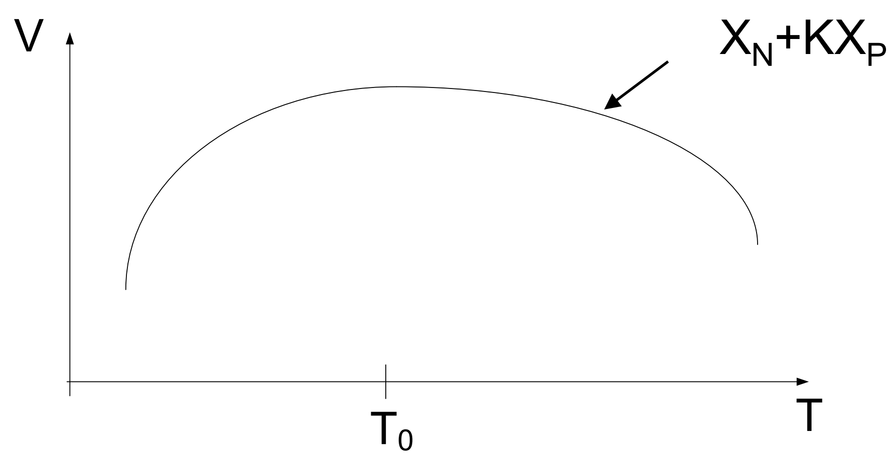
Pick K so that at some temperature T_0, $\left.\frac{\partial(\mathbf{X_N + KX_P})}{\partial \mathbf{T}}\right|_{\mathbf{T=T_0}} = \mathbf{0}$

Standard Approach to Building Voltage References



Standard Approach to Building Voltage References





Select K so that

$$\left.\frac{\partial\left(X_N + KX_P\right)}{\partial T}\right|_{T=T_0} = 0$$

Bandgap Voltage References

Consider two BJTs (or diodes)

$$I_C(T) = \left(\tilde{I}_{SX}\left[T^m e^{\frac{-V_{G0}}{V_t}}\right]\right) e^{\frac{V_{BE}(T)}{V_t}}$$ .... Exponential form

$$V_{BE} = V_{G0} + V_t \ln\left(\frac{I_C}{\tilde{J}_{SX} A_E}\right) - mV_t \ln T$$ .... Logarithmic form

$$V_{BE2} - V_{BE1} = \Delta V_{BE} = \left[\frac{k}{q}\ln\left(\frac{I_{C2}}{I_{C1}}\frac{A_{E1}}{A_{E2}}\right)\right]T$$

If the $\frac{I_{C2}A_{E1}}{I_{C1}A_{E2}}$ ratio is constant and >1, the TC of ΔV_{BE} is positive

ΔV_{BE} is termed a PTAT voltage (Proportional to Absolute Temperature)

This relationship applies irrespective of how temperature dependent I_{C1} and I_{C2} may be provided the ratio is constant !!

Bandgap Voltage References

Consider two BJTs (or diodes)

$$V_{BE2} - V_{BE1} = \Delta V_{BE} = \left[\frac{k}{q}\ln\left(\frac{I_{C2}A_{E1}}{I_{C1}A_{E2}}\right)\right]T$$

$$\frac{\partial\left(V_{BE2} - V_{BE1}\right)}{\partial T} = \frac{k}{q}\ln\left(\frac{I_{C2}A_{E1}}{I_{C1}A_{E2}}\right)$$

At room temperature if $\ln\left(\frac{I_{C2}A_{E1}}{I_{C1}A_{E2}}\right) = 1$

$$V_{BE2} - V_{BE1} = \left[8.6x10^{-5}x300\right] = 25.8mV$$

and

$$\left.\frac{\partial\left(V_{BE2} - V_{BE1}\right)}{\partial T}\right|_{T=T_0=300^\circ K} = 8.6x10^{-5} = 86\mu V/^\circ C$$

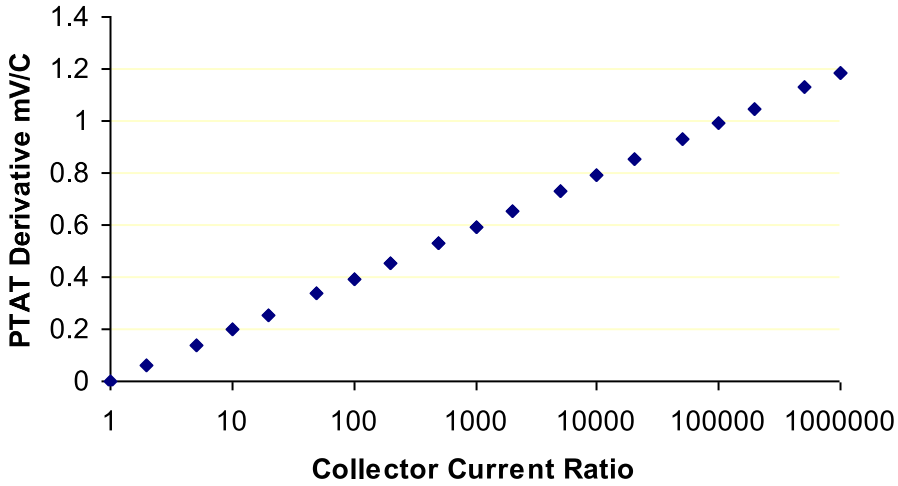
The temperature coefficient of the PTAT voltage is rather small

Bandgap Voltage References

Consider two BJTs (or diodes)

$$\frac{\partial(V_{BE2}-V_{BE1})}{\partial T}=\frac{k}{q}\ln\left(\frac{I_{C2}}{I_{C1}}\right)$$

At room temperature if $A_{E1}=A_{E2}$



The temperature coefficient of the PTAT voltage is rather small even if large collector current ratios are used

Bandgap Voltage References

Consider two BJTs (or diodes)   Typically, $m=2.3$, $V_{G0}=1.2V$ Assume $V_{BE}\approx 0.65V$

$$I_C(T) = \left(\tilde{J}_{SX} A_E \left[T^m e^{\frac{-V_{G0}}{V_t}}\right]\right) e^{\frac{V_{BE}(T)}{V_t}}$$

$$V_{BE}=V_{G0}+V_t \ln\left(\frac{I_C}{\tilde{J}_{SX} A_E}\right)-mV_t \ln T$$

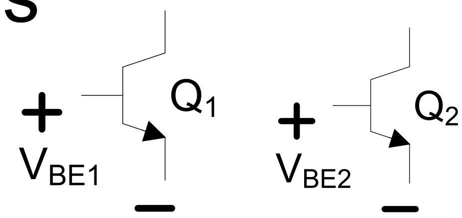
If I_C is independent of temperature, it follows that

$$\frac{\partial V_{BE}}{\partial T} = \frac{k}{q}\left[-m+\left(\frac{V_{BE}-V_{G0}}{V_t}\right)\right]$$

$$\left.\frac{\partial V_{BE}}{\partial T}\right|_{T=T_0=300^\circ K} \cong 8.6x10^{-5}\left[-2.3+\left(\frac{0.65-1.2}{25mV}\right)\right] \cong -2.1mV/^\circ C$$

Bandgap Voltage References

Consider two BJTs (or diodes)

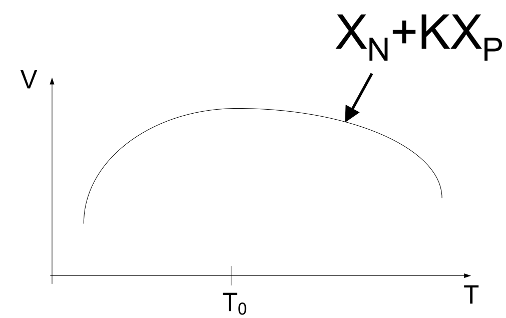


Typically, $m=2.3$, $V_{G0}=1.2V$ Assume $V_{BE}\approx 0.65V$

Thus if I_C independent of temperature and if $\ln\left(\frac{I_{C2}A_{E1}}{I_{C1}A_{E2}}\right)=1$

$$\left.\frac{\partial V_{BE}}{\partial T}\right|_{T=T_0=300^oK} \cong -2.1mV/^{o}C$$

$$\left.\frac{\partial\left(V_{BE2}-V_{BE1}\right)}{\partial T}\right|_{T=T_0=300^oK} = 86\mu V/^{o}C$$



Magnitude of TC of PTAT source is much smaller than that of V_{BE} source

Define:

$X_N=V_{BE}$ $X_P=V_{BE2}-V_{BE1}$

Create circuit with:

$$\mathbf{X_{OUT} = X_N + KX_P}$$

If we want $\left.\frac{\partial(\mathbf{X_N+KX_P})}{\partial \mathbf{T}}\right|_{\mathbf{T=T_0}} = \mathbf{0}$ K will need to be large

Bandgap Voltage References

Consider two BJTs (or diodes)

$$V_{BE} = V_{G0} + V_t \ln\left(\frac{I_C}{\tilde{J}_{SX} A_E}\right) - mV_t \ln T$$

It was just shown that if I_C is independent of temperature

$$\left.\frac{\partial V_{BE}}{\partial T}\right|_{T=T_0=300^{\circ}K} \cong 8.6\text{x}10^{-5}\left[-2.3+\left(\frac{0.65-1.2}{25mV}\right)\right] \cong -2.1mV/^{\circ}C$$

If I_C is reasonably independent of temperature, V_{BE} will still provide a negative TC

Even if I_C is highly dependent on temperature, V_{BE} will still provide a negative TC

Observe V_{G0} appears prominently in V_{BE}

Bandgap Voltage References

Consider two BJTs (or diodes)

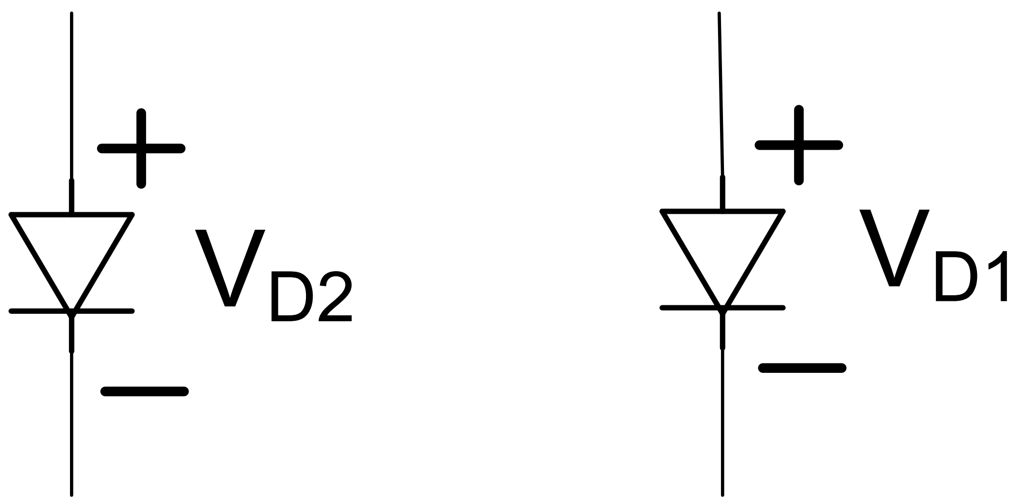
Key observation about diodes and diode-connected BJTs

1. If ratio of currents in two devices is constant, ΔV_{BE} can be PTAT independent of the temperature dependence of the currents and sensitivity of ΔV_{BE} to T is small

2. VBE has a negative temperature coefficient for a wide range of temperature dependent or temperature independent currents and sensitivity is much larger than that of ΔV_{BE}

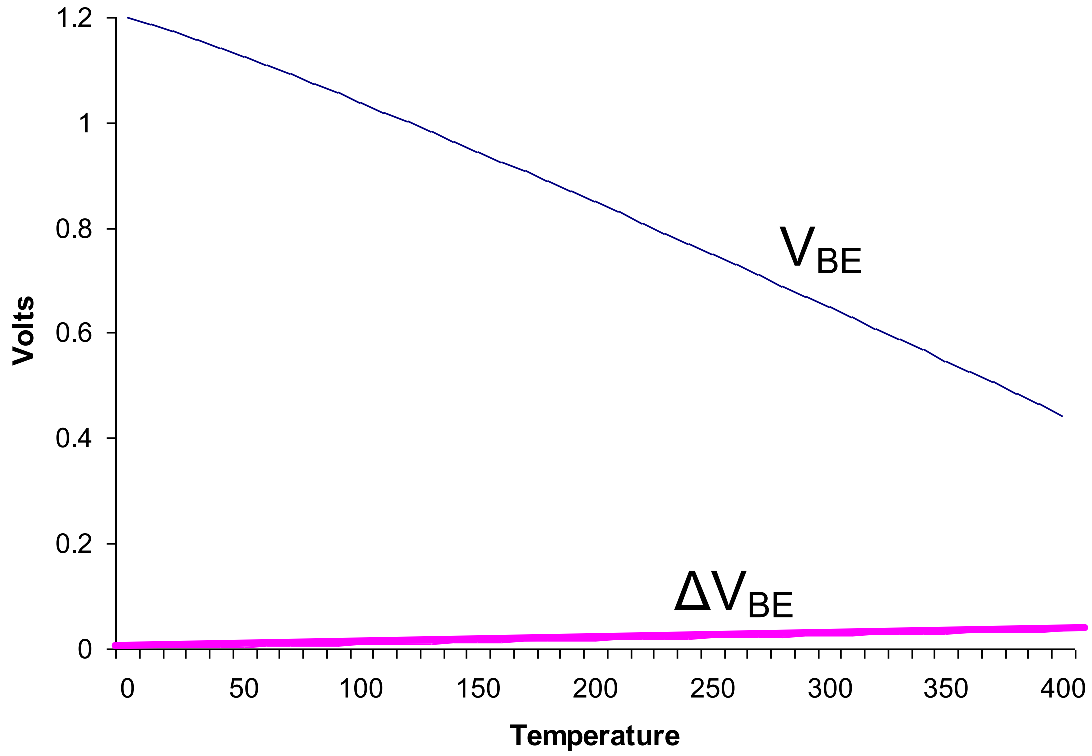
Bandgap Reference Circuits

- Circuits that implement ΔV_{BE} and V_{BE} or ΔV_D and V_D widely used to build bandgap references



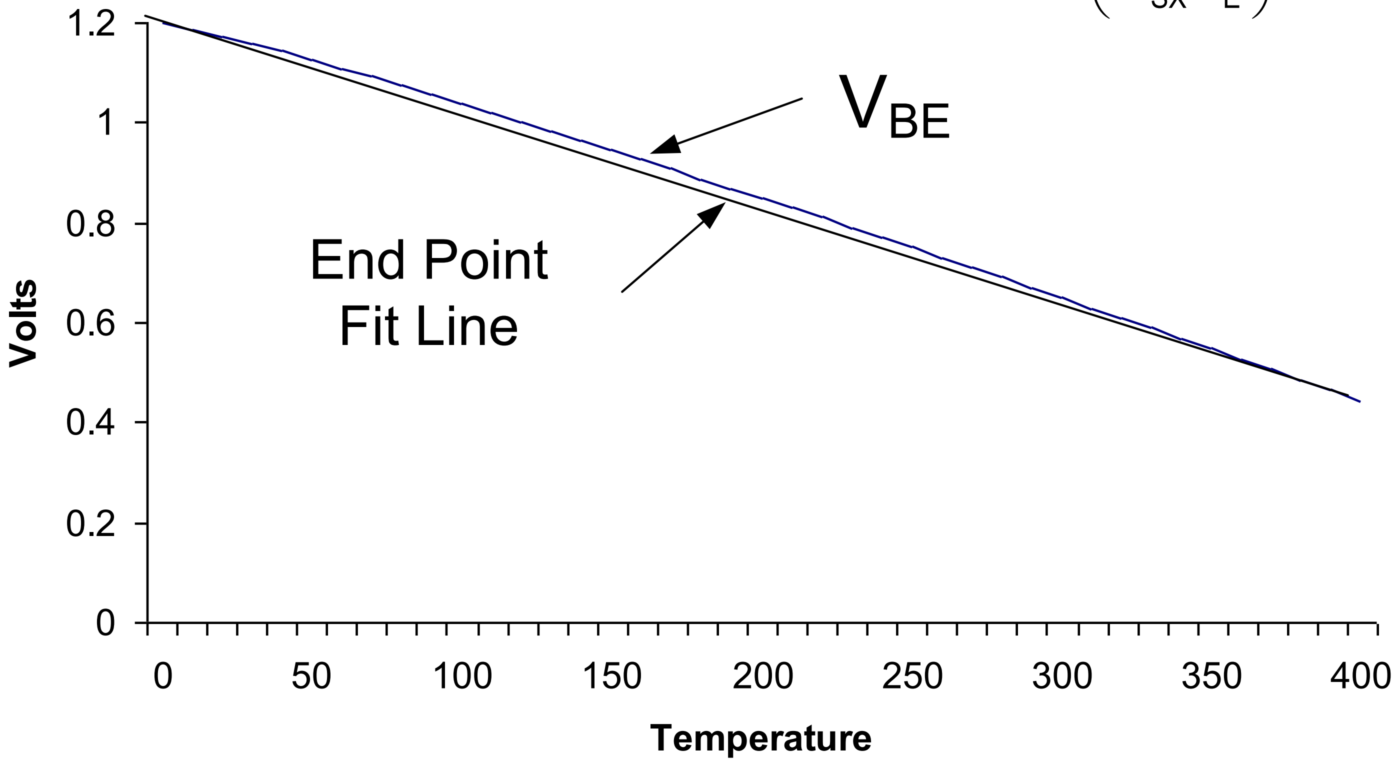
V_{BE} and ΔV_{BE} with constant I_C

$$V_{BE} = V_{G0} + V_t \ln\left(\frac{I_C}{\tilde{J}_{SX} A_E}\right) - m V_t \ln T$$



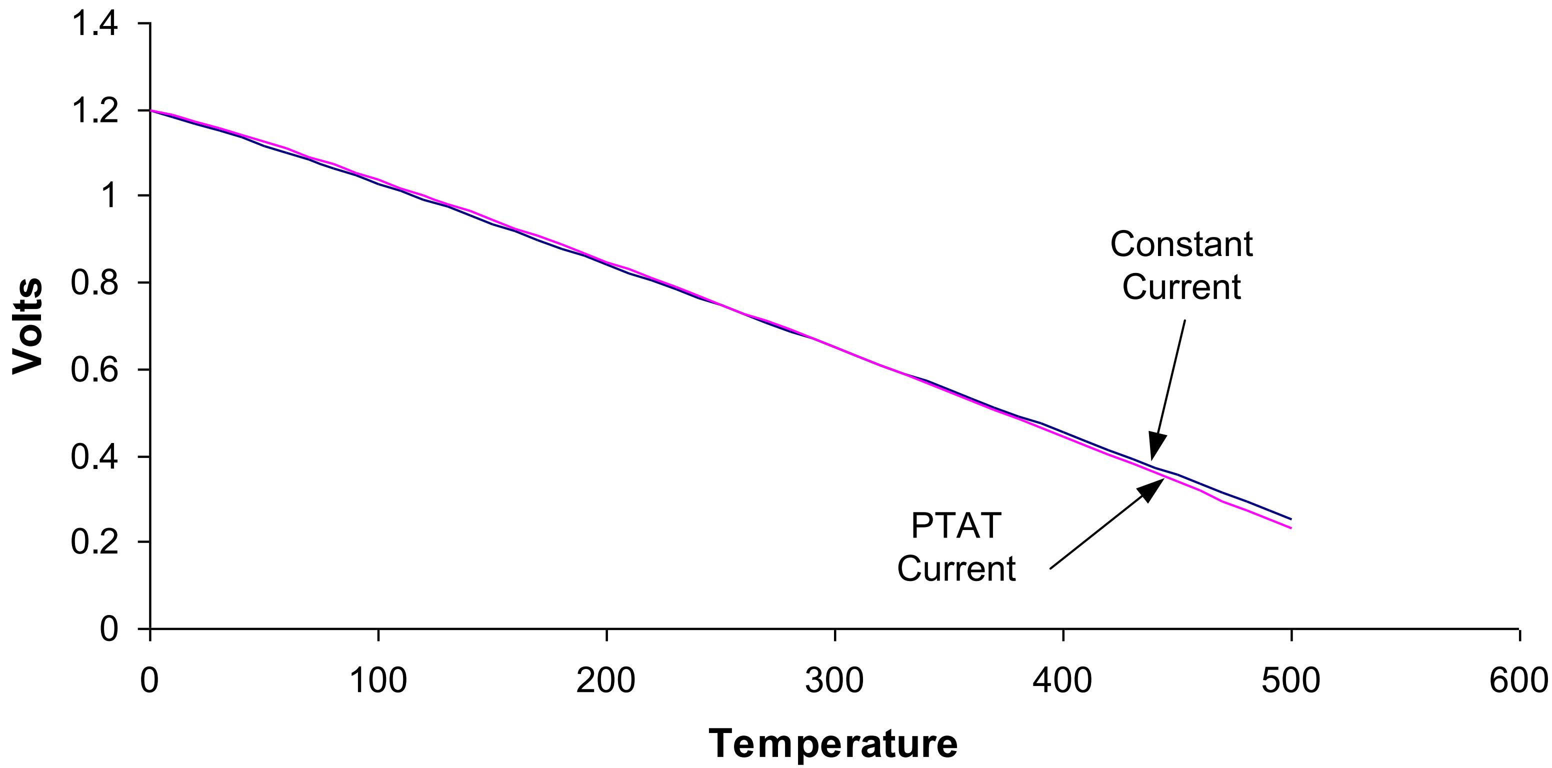
V_{BE} plot for constant I_C

$$V_{BE} = V_{G0} + V_t \ln\left(\frac{I_C}{\tilde{J}_{SX} A_E}\right) - mV_t \ln T$$



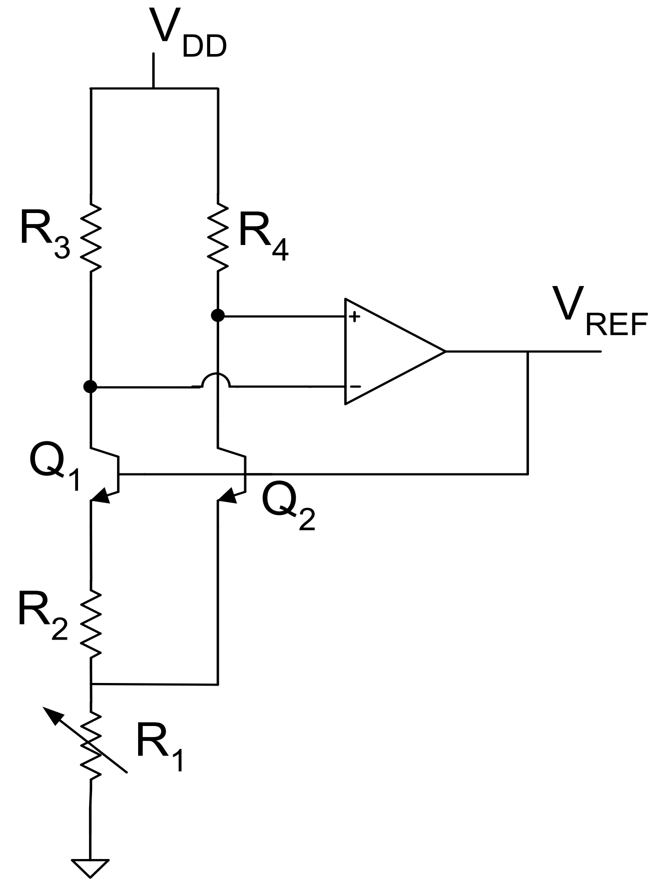
Combined effects of the T and TlnT terms in V_{BE} is nearly linear dependent on T

Comparison of V_{BE} with constant current and PTAT current



Even if I_C is highly-dependent on current, temperature dependence of V_{BE} is still nearly linearly dependent upon T

Early Bandgap Reference (and still widely used!)



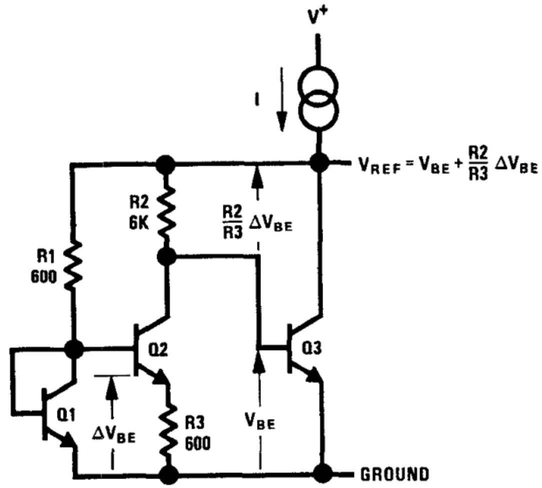
P.Brokaw, “A Simple Three-Terminal IC Bandgap Reference”, IEEE Journal of Solid State Circuits, Vol. 9, pp. 388-393, Dec. 1974.

- Brokaw coined term “bandgap reference” when referring to this circuit
- Properties very similar circuits introduced by Widlar and Kujik a small while earlier
- Paper submitted May 1974, Widlar paper submitted March 1970

IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. SC-6, NO 1, FEBRUARY 1971

New Developments in IC Voltage Regulators

ROBERT J. WIDLAR



Widlar retired in Dec. 1970 at the age of 33

Widlar observed ΔV_{BE} is PTAT in 1965

] R. J. Widlar, "Some circuit design techniques for linear integrated circuits," *IEEE Trans. Circuit Theory*, vol. CT-12, pp. 586–590, December 1965.

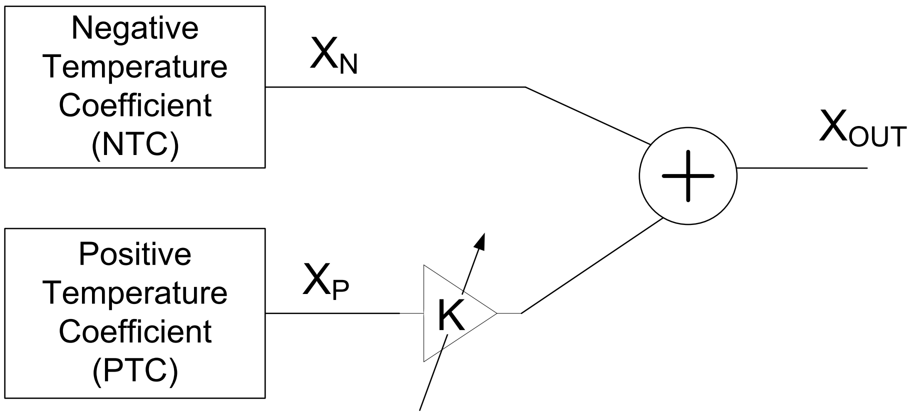
Most Published Analysis of Bandgap Circuits

V_{REF} often expressed as:

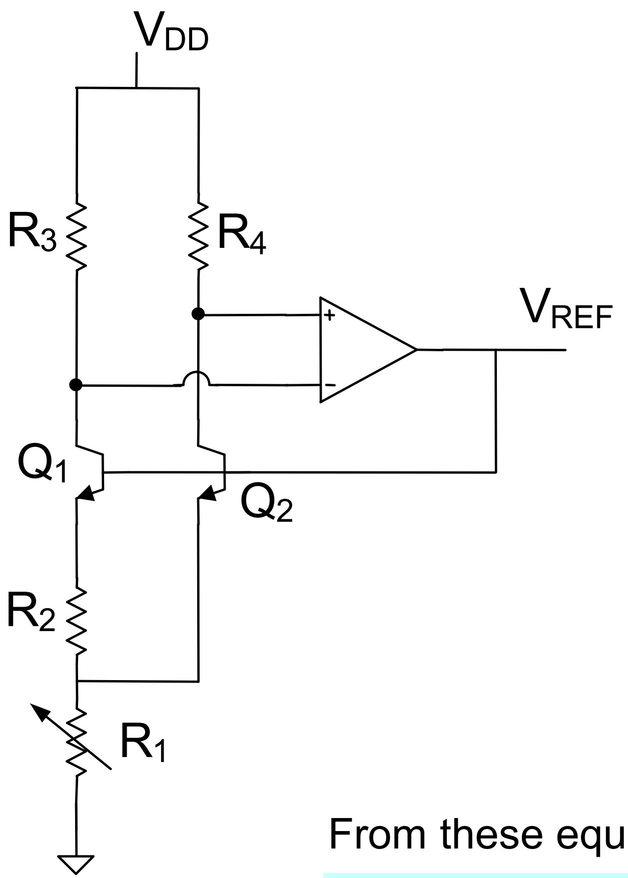
$$V_{REF} = V_{G0} + \frac{T}{T_0}\left(V_{BE0} - V_{G0}\right) + K\frac{kT}{q}\ln\left(\frac{J_2}{J_1}\right) + (m-1)\frac{kT}{q}\ln\left(\frac{T_0}{T}\right)$$

where K is the gain of the PTAT signal

(Not a solution and dependent upon both T_0 and V_{BE0})



First Bandgap Reference (and still widely used!)



$$I_{E1}R_2 + V_{BE1} = V_{BE2}$$

$$V_{REF} = V_{BE2} + \left(I_{E1} + I_{E2}\right)R_1$$

$$I_{C1} = \frac{V_{DD} - V_{C2}}{R_3}$$

$$I_{C2} = \frac{V_{DD} - V_{C2}}{R_4}$$

$$I_{C1} = \alpha_1 I_{E1}$$

$$I_{C2} = \alpha_2 I_{E2}$$

$$I_{E1} = I_{E2}\left[\frac{\alpha_2}{\alpha_1}\frac{R_4}{R_3}\right] \rightarrow I_{C1} = I_{C2}\left[\frac{R_4}{R_3}\right]$$

$$\alpha = \frac{\beta}{1+\beta}$$

From these equations can show

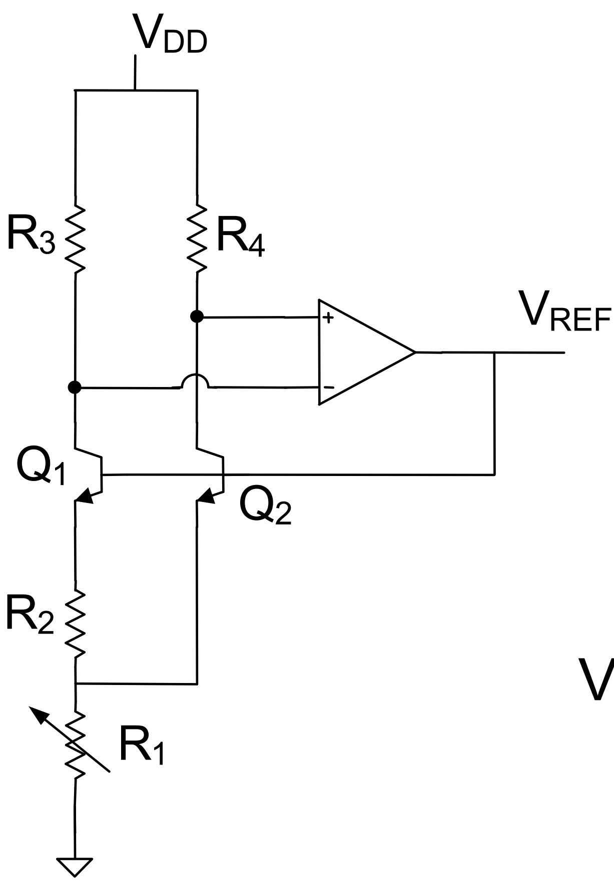
$$V_{REF} = V_{BE2} + \left(V_{BE2} - V_{BE1}\right)\left[\frac{R_1}{R_2}\left(1 + \frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]$$

Not a solution but can provide zero temp slope by adjusting R_1

First Bandgap Reference (and still widely used!)

Will now obtain solution for V_{REF} (in terms of component values and model parameters)

$$V_{REF} = V_{BE2} + \left(V_{BE2} - V_{BE1}\right)\left[\frac{R_1}{R_2}\left(1+\frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]$$



$$V_{BE1} = V_{G0} + V_t \ln\left(\frac{I_{C1}}{\tilde{J}_{SX} A_{E1}}\right) - mV_t \ln T$$

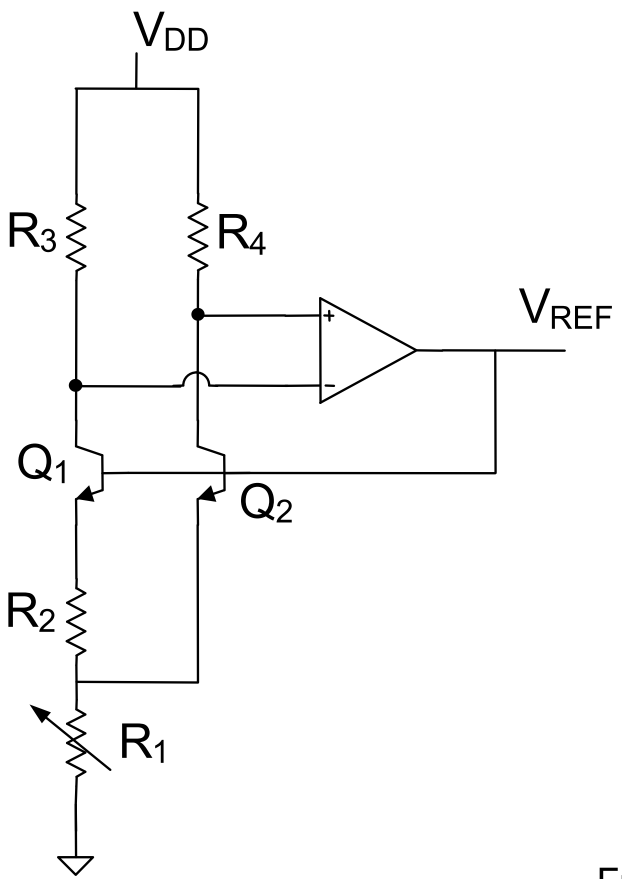
$$V_{BE2} = V_{G0} + V_t \ln\left(\frac{I_{C2}}{\tilde{J}_{SX} A_{E2}}\right) - mV_t \ln T$$

$$I_{C1} = I_{C2}\left[\frac{R_4}{R_3}\right]$$

$$V_{BE2} - V_{BE1} = \Delta V_{BE} = \left[\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\left[\frac{R_3}{R_4}\right]\right)\right]T$$

First Bandgap Reference (and still widely used!)

Will now obtain solution for V_{REF} (in terms of component values and model parameters)



$$V_{REF} = V_{BE2} + \left(V_{BE2} - V_{BE1}\right)\left[\frac{R_1}{R_2}\left(1+\frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]$$

$$V_{BE1}=V_{G0}+V_t \ln\left(\frac{I_{C1}}{\tilde{J}_{SX}A_{E1}}\right)-mV_t \ln T$$

$$V_{BE2}=V_{G0}+V_t \ln\left(\frac{I_{C2}}{\tilde{J}_{SX}A_{E2}}\right)-mV_t \ln T$$

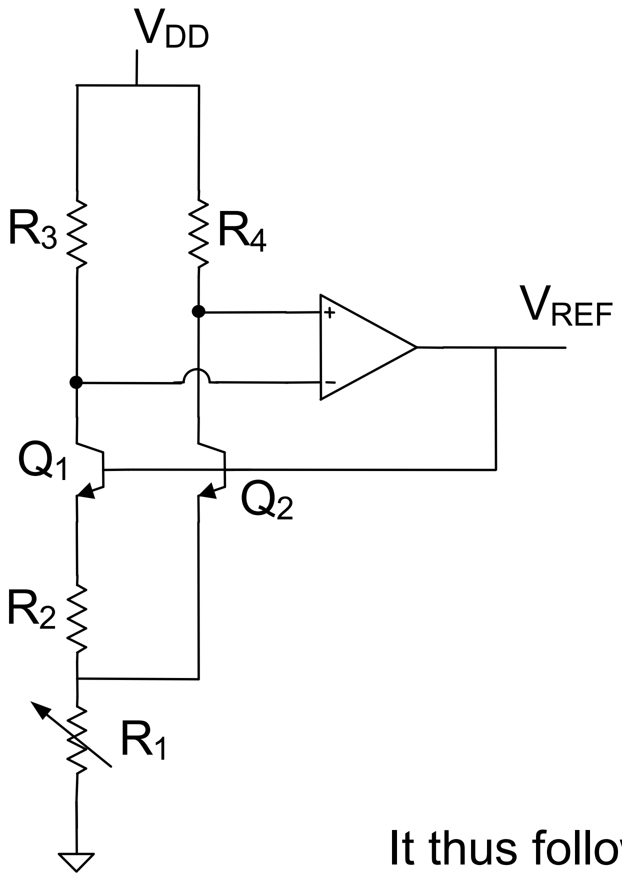
$$I_{C1}=I_{C2}\left[\frac{R_4}{R_3}\right]$$

$$\frac{I_{C1}}{\alpha_1}R_2+V_{BE1}=V_{BE2}$$

From the expression for V_{BE2} and some routine but tedious manipulations it follows that

$$V_{BE2}=V_{G0}+(1-m)V_t \ln T + V_t \ln\left(\frac{k}{q}\frac{\alpha_1}{R_2 A_{E2}\tilde{J}_{SX}}\frac{R_3}{R_4}\ln\left(\frac{A_{E1}}{A_{E2}}\frac{R_3}{R_4}\right)\right)$$

First Bandgap Reference (and still widely used!)



$$V_{REF} = V_{BE2} + \left(V_{BE2} - V_{BE1}\right)\left[\frac{R_1}{R_2}\left(1 + \frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]$$

$$V_{BE2} - V_{BE1} = \left[\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\left[\frac{R_3}{R_4}\right]\right)\right]T$$

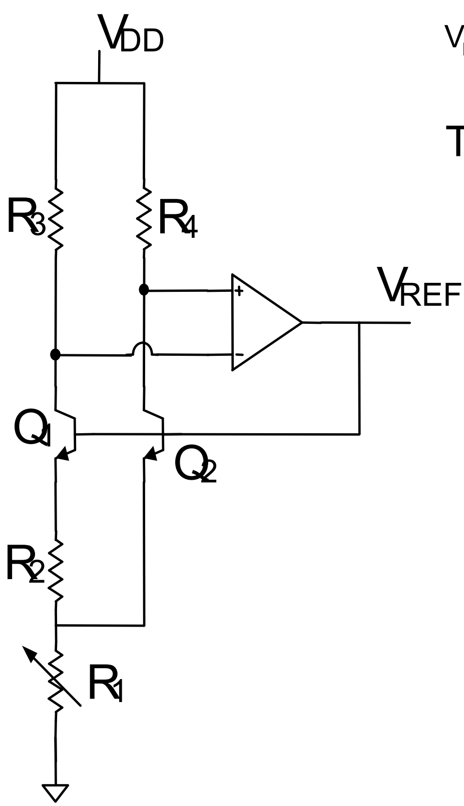
$$V_{BE2} = V_{G0} + (1-m)V_t \ln T \; + V_t \ln\left(\frac{k}{q}\frac{\alpha_1}{R_2 A_{E2}\tilde{J}_{SX}}\frac{R_3}{R_4}\ln\left(\frac{A_{E1}}{A_{E2}}\frac{R_3}{R_4}\right)\right)$$

It thus follows that:

$$V_{REF} = V_{G0} + V_t \ln\left\{\frac{\alpha_1}{R_2}\frac{R_3}{R_4}T\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\frac{R_3}{R_4}\right)\right\} - V_t\left(\ln\left(\tilde{I}_{SX2}\right) + m\ln T\right) + \left[\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\left(\frac{R_3}{R_4}\right)\right)\right]\left[\frac{R_1}{R_2}\left(1 + \frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]T$$

First Bandgap Reference (and still widely used!)

$$V_{REF} = V_{BE2} + \left(V_{BE2} - V_{BE1}\right)\left[\frac{R_1}{R_2}\left(1 + \frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]$$



$$V_{REF} = V_{G0} + V_t \ln\left\{\frac{\alpha_1}{R_2}\frac{R_3}{R_4} T \frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\frac{R_3}{R_4}\right)\right\} - V_t\left(\ln\left(\tilde{I}_{SX2}\right) + m\ln T\right) + \left[\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\left(\frac{R_3}{R_4}\right)\right)\right]\left[\frac{R_1}{R_2}\left(1 + \frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]T$$

This can be expressed after some tedious algebraic manipulations as

$$V_{REF} = a_1 + b_1 T + c_1 T\ln T$$

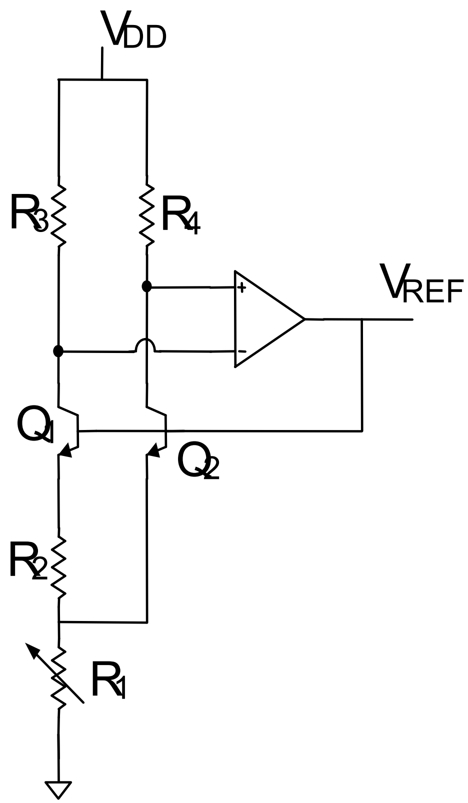
where

$$\mathbf{a_1 = V_{GO}}$$

$$b_1 = \frac{k}{q}\left(\frac{R_1}{R_2}\left(1 + \frac{R_3\alpha_1}{R_4\alpha_2}\right)\ln\left(\frac{R_3}{R_4}\frac{A_{E1}}{A_{E2}}\right) + \ln\left(\frac{k}{q}\frac{R_3}{R_4}\alpha_1\frac{\ln\left(\frac{R_3}{R_1}\frac{A_{E1}}{A_{E2}}\right)}{\tilde{I}_{SK2}R_2}\right)\right)$$

$$c_1 = \frac{k}{q}(1 - m)$$

First Bandgap Reference (and still widely used!)



$$V_{REF} = a_1 + b_1 T + c_1 T \ln T$$

$$\mathbf{a_1 = V_{GO}}$$

$$b_1 = \frac{k}{q}\left(\frac{R_1}{R_2}\left(1+\frac{R_3\alpha_1}{R_4\alpha_2}\right)\ln\left(\frac{R_3}{R_4}\frac{A_{E1}}{A_{E2}}\right) + \ln\left(\frac{k}{q}\frac{R_3}{R_4}\alpha_1\frac{\ln\left(\frac{R_3}{R_1}\frac{A_{E1}}{A_{E2}}\right)}{\tilde{I}_{SK2}R_2}\right)\right)$$

$$c_1 = \frac{k}{q}(1-m)$$

$$\frac{dV_{REF}}{dT} = b_1 + c_1(1+\ln T) = 0$$

$$T_{INF} = e^{-\left(1+\frac{b_1}{c_1}\right)}$$

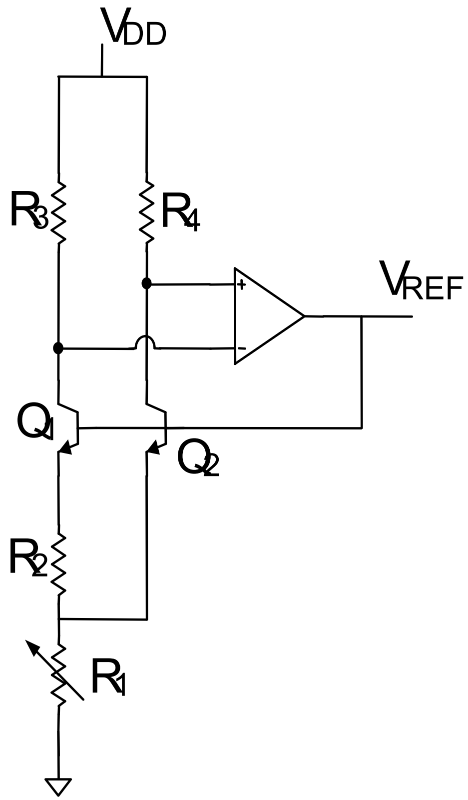
$$b_1 = -c_1(1+\ln T_{INF})$$

at T_{INF} $\quad V_{REF} = a_1 - c_1 T_{INF}$

$$V_{REF} = V_{G0} + \frac{kT_{INF}}{q}(m-1)$$

$\frac{kT_{INF}}{q}(m-1)$ is small ⇒ Nearly V_{GO} output at T_{INF}

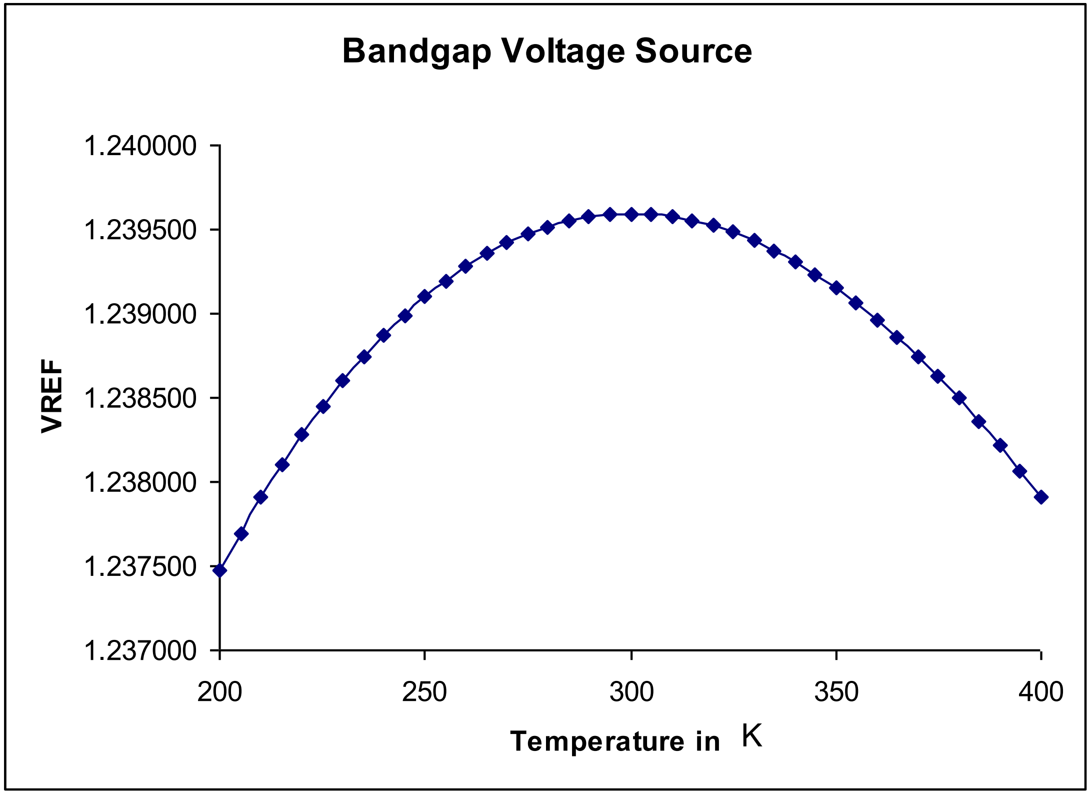
First Bandgap Reference (and still widely used!)



$$V_{REF} = a_1 + b_1 T + c_1 T \ln T$$

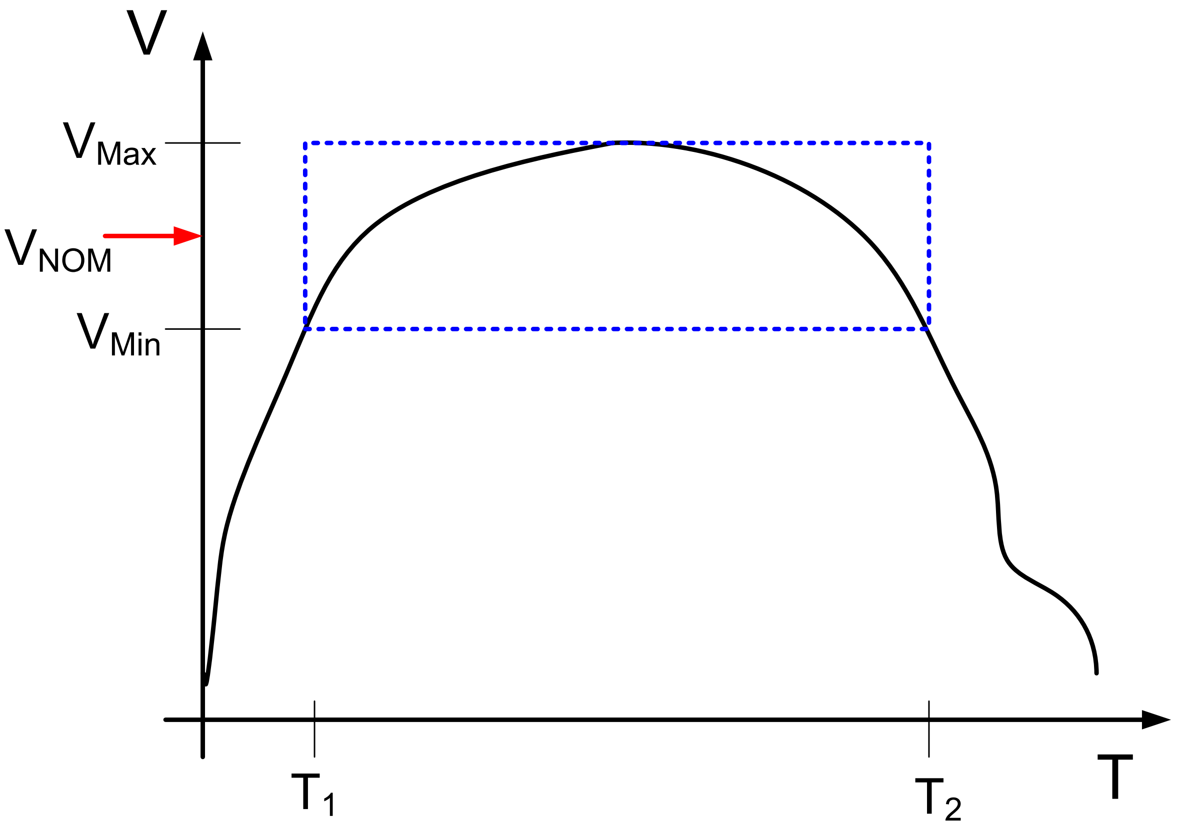
$$V_{REF}(T_{INF}) = V_{G0} + \frac{kT_{INF}}{q}(m-1)$$

VGO	1.206
TO	300
VBEO2	0.65
m-1	1.3
k/q	8.61E-05



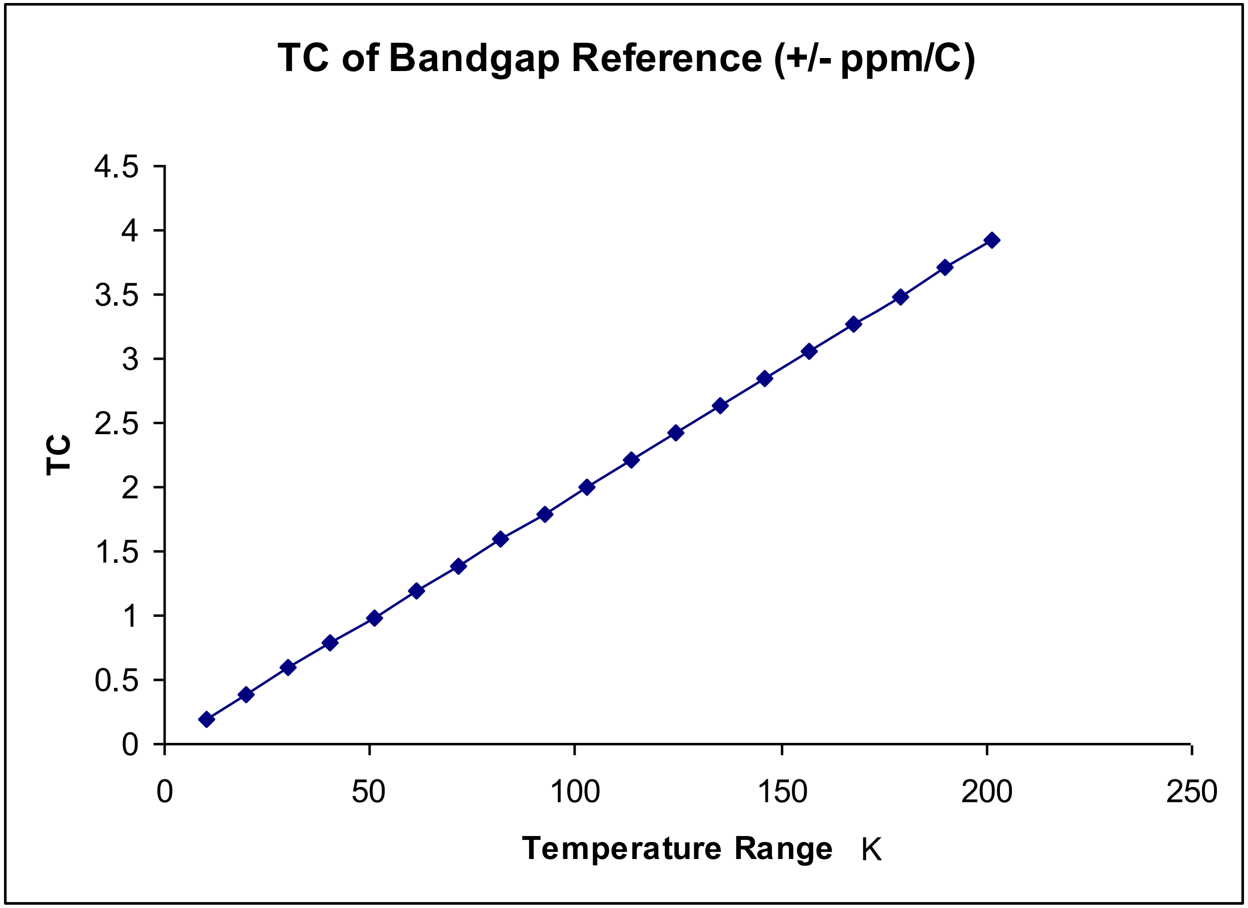
Only 2mV change over 200°C temp range !

Temperature Coefficient



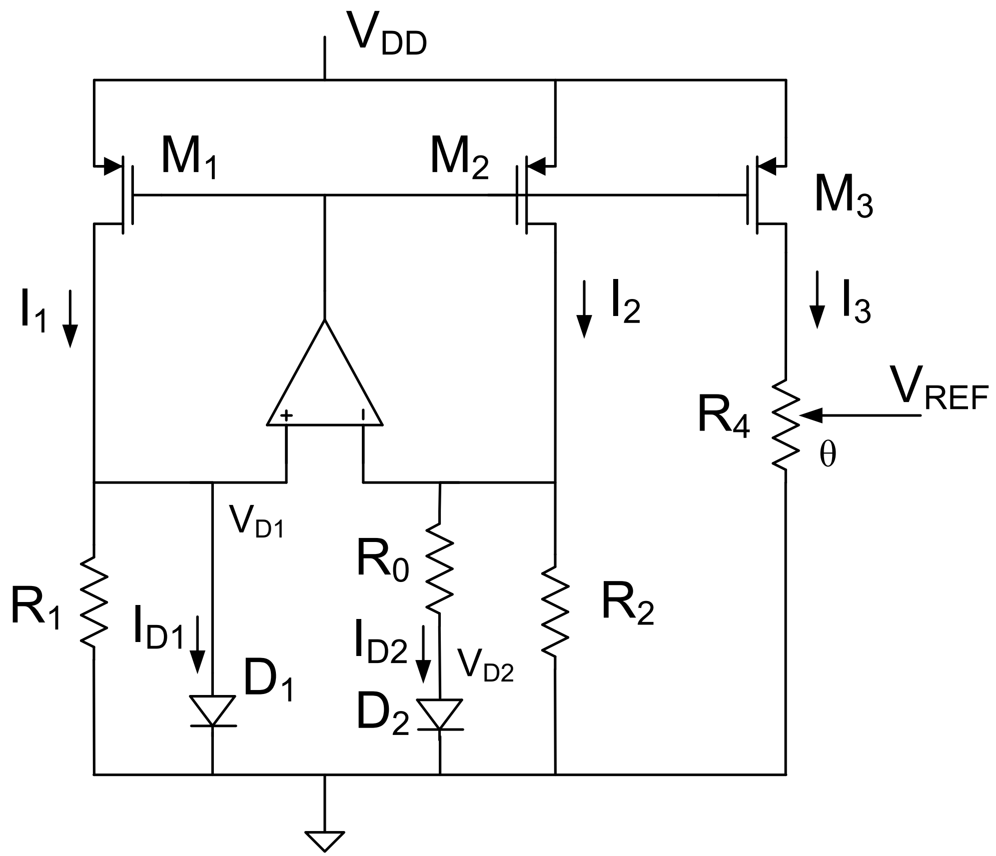
$$TC = \frac{V_{MAX} - V_{MIN}}{T_2 - T_1}$$

$$TC_{ppm} = \frac{V_{MAX} - V_{MIN}}{V_{NOM}(T_2 - T_1)} 10^6$$


TC of Bandgap Reference (+/- ppm/C)
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
TC
0
50
100
150
200
250
Temperature Range K

Banba Bandgap Reference

$$\frac{I_{D2}}{I_{D1}} = \text{constant}$$



[7] H. Banba, H. Shiga, A. Umezawa, T. Miyaba, T. Tanzawa, A. Atsumi, and K. Sakkui, IEEE Journal of Solid-State Circuits, Vol. 34, pp. 670-674, May 1999.

Note this was introduced 25 years after the Brokaw reference

Bamba Bandgap Reference

$$I_{R0} = \frac{\Delta V_{BE}}{R_0}$$

$$I_{R1} = \frac{V_{BE1}}{R_1}$$

$$I_{R2} = I_{R1}$$

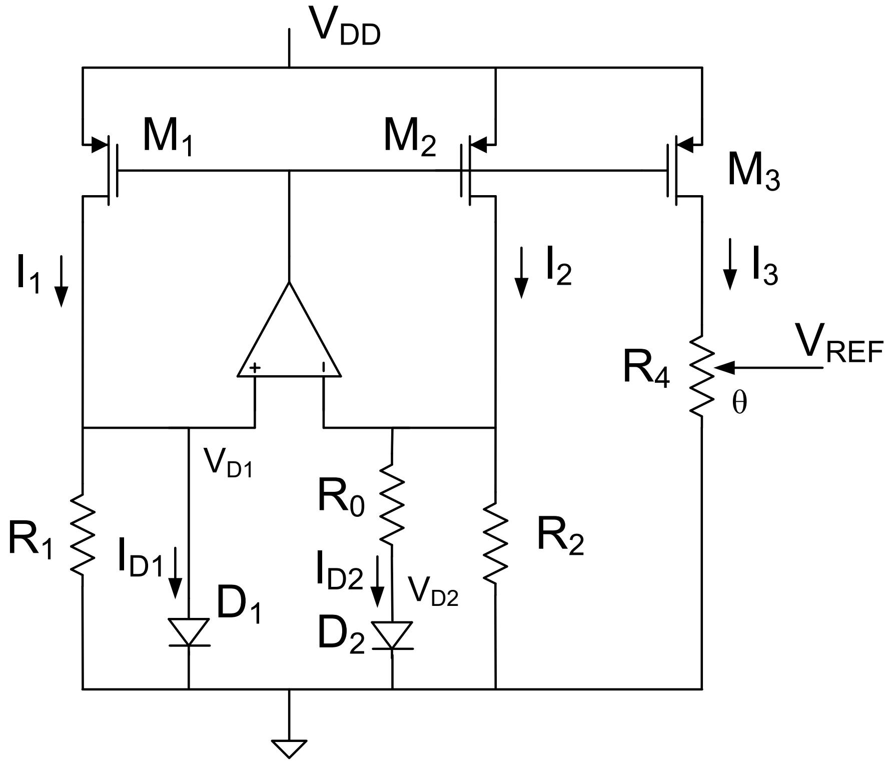
$$I_2 = I_{R0} + I_{R2}$$

$$I_3 = KI_2$$

K is the ratio of I_3 to I_2

$$V_{REF} = \theta I_3 R_4$$

$$\frac{I_{D2}}{I_{D1}} = \text{constant}$$



Substituting, we obtain

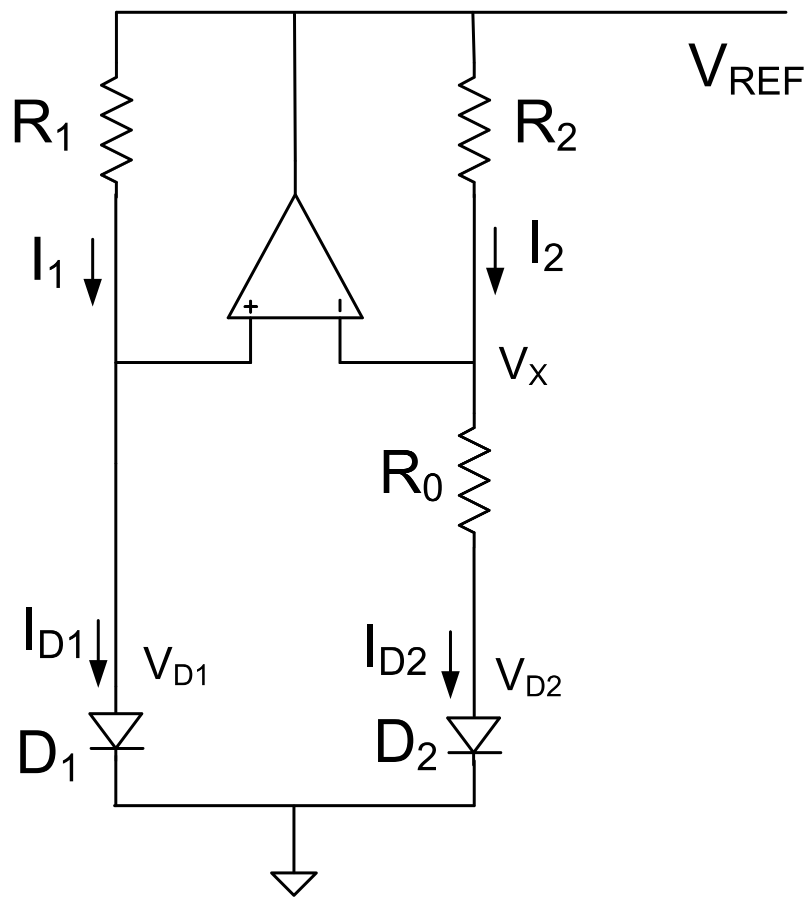
$$V_{REF} = \theta K R_4 \left(\frac{V_{BE}}{R_1} + \frac{\Delta V_{BE}}{R_0} \right) \rightarrow V_{REF} = \theta K \frac{R_4}{R_1} \left(V_{BE} + \frac{R_1}{R_0} \Delta V_{BE} \right)$$

With some tedious algebra, it follows that $V_{REF} = a_{11} + b_{11}T + c_{11}T\ln T$

Note this is of the same form as that of the Brokow reference !

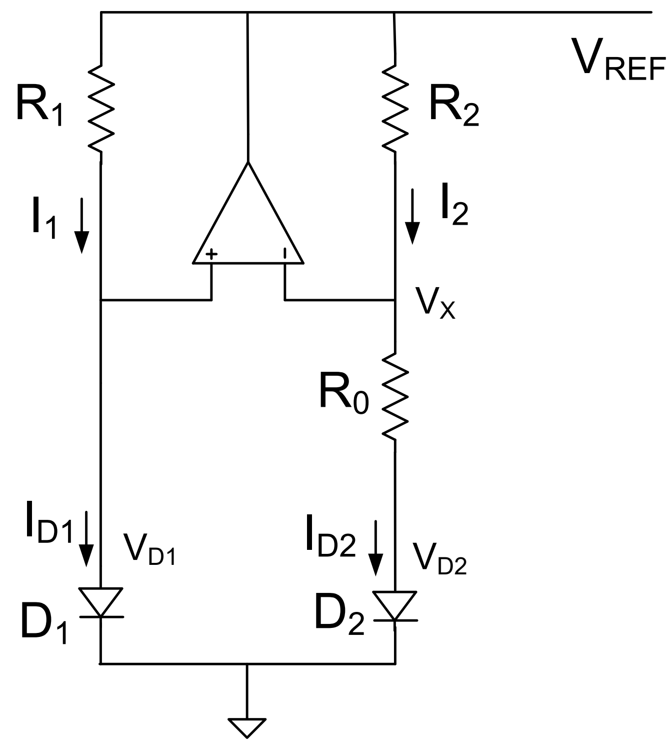
Kujik Bandgap Reference

$$\frac{I_{D2}}{I_{D1}} = \text{constant}$$



[12] K. Kuijk, “A Precision Reference Voltage Source”, IEEE Journal of Solid State Circuits, Vol. 8, pp. 222-226, June 1973.

Kujik Bandgap Reference



$$I_{R0} = \frac{\Delta V_{BE}}{R_0}$$

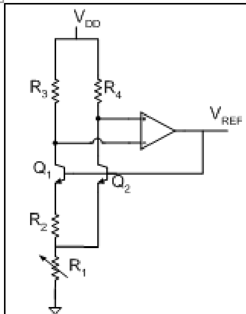
$$I_2 = I_{R0}$$

$$V_{REF} = I_2 R_2 + V_{BE1}$$

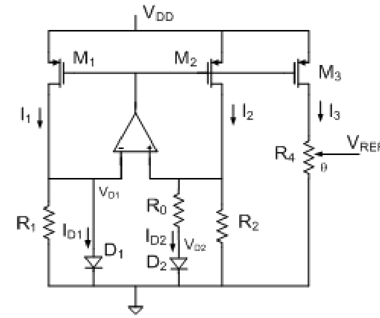
solving, we obtain

$$V_{REF} = \frac{R_2}{R_0} \Delta V_{BE} + V_{BE1}$$

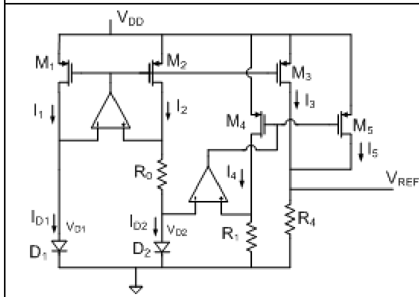
$$V_{REF} = a_{22} + b_{22} T + c_{22} T \ln T$$



Brokow



Banba



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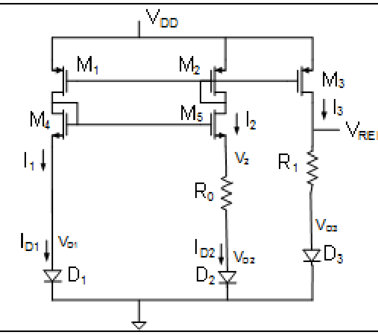

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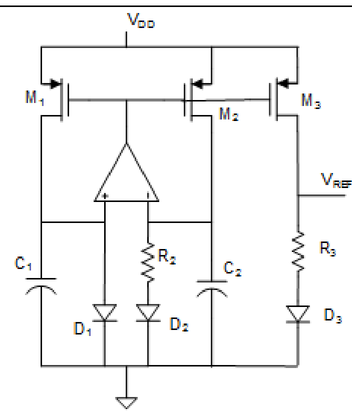
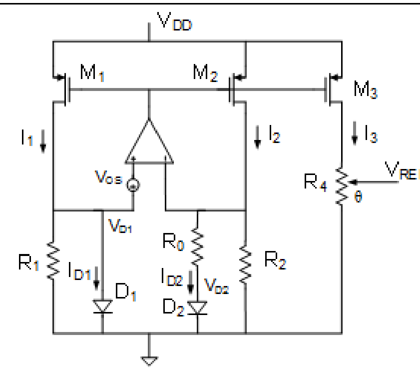


Modified Mietus



Modified Banba

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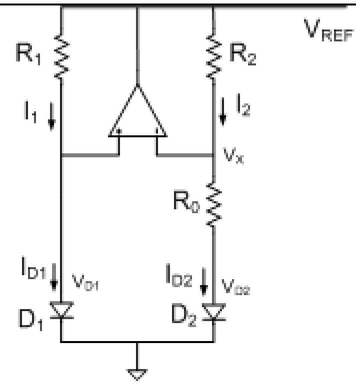
<td>
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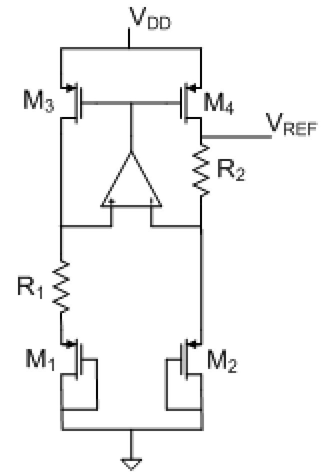
<td>
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<td>Kuijk</td>
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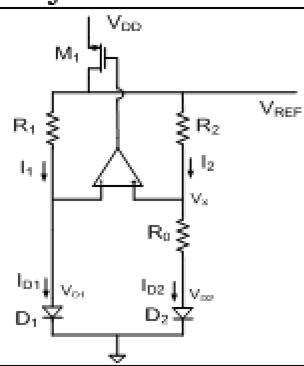
<td>
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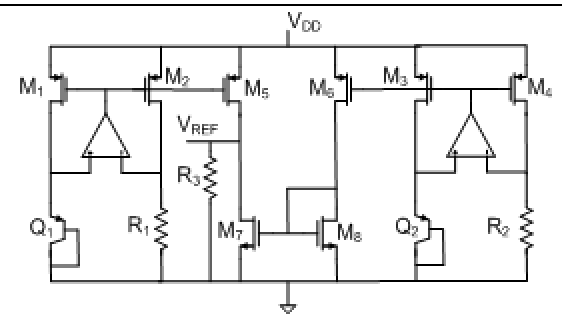
<td>
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<td>Modified Kuijk</td>
<td>Zhu</td>
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Almost all of the published bandgap references have an output of the form:

$$V_{REF} = a + bT + cT \ln T$$

	a	b	c
Brokow	$a_1 = V_{G0}$	$b_1 = \frac{k}{q}\left(\frac{R_1}{R_2}\left(1+\frac{R_3\alpha_1}{R_4\alpha_2}\right)\ln\left(\frac{R_3}{R_4}\frac{A_{S1}}{A_{S2}}\right)+\ln\left(\frac{k}{q}\frac{R_3}{R_4}\alpha_1\frac{\ln\left(\frac{R_3}{R_1}\frac{A_{S1}}{A_{S2}}\right)}{\tilde{I}_{SX2}R_2}\right)\right)$	$c_1 = \frac{k}{q}(1-m)$
Banba	$a_2 = \left[\frac{R_4}{R_1}\theta K_3\right]V_{G0}$	$b_2 = \left[\frac{k}{q}\theta K_3\right]\left(\frac{R_4}{R_0}\ln\left(\frac{A_{D2}}{A_{D1}}\right)+\frac{R_4}{R_1}\ln\left(\frac{k}{q}\frac{\ln\left(\frac{A_{D2}}{A_{D1}}\right)}{R_0A_{D1}\tilde{J}_{SX1}}\right)\right)$	$c_2 = \left[\frac{R_4}{R_1}\theta K_3\right]\frac{k}{q}(1-m)$
Mieteus	$a_3 = K_5V_{G0}$	$b_3 = \frac{k}{q}\left(K_3\frac{R_4}{R_0}\ln\left(K_1\frac{A_{D2}}{A_{D1}}\right)+K_5\left(\ln\frac{k}{q}+\frac{\ln\left(K_1\frac{A_{D2}}{A_{D1}}\right)}{J_{SX}A_{D2}}\right)\right)$	$c_3 = \frac{k}{q}K_5(1-m)$
Kujik	$a_4 = V_{G0}$	$b_4 = \frac{k}{q}\left[\frac{R_2}{R_0}\ln\left(\frac{R_2}{R_1}\frac{A_{D2}}{A_{D1}}\right)+\ln\left(\frac{R_2}{R_1}\frac{k}{q}\frac{\ln\left(\frac{R_2}{R_1}\frac{A_{D2}}{A_{D1}}\right)}{R_0A_{D1}\tilde{J}_{SX}}\right)\right]$	$c_4 = \frac{k}{q}(1-m)$
Modified Kuijk	$a_5 = V_{G0}$	$b_5 = \frac{k}{q}\left[\frac{R_2}{R_0}\ln\left(\frac{R_2}{R_1}\frac{A_{D2}}{A_{D1}}\right)+\ln\left(\frac{R_2}{R_1}\frac{k}{q}\frac{\ln\left(\frac{R_2}{R_1}\frac{A_{D2}}{A_{D1}}\right)}{R_0A_{D1}\tilde{J}_{SX}}\right)\right]$	$c_5 = \frac{k}{q}(1-m)$
Modified Kuijk	$a_6 = KV_{G0}$	$b_6 = \frac{k}{q}K\left[\frac{R_2}{R_0}\ln\left(\frac{R_2}{R_1}\frac{A_{D2}}{A_{D1}}\right)+\ln\left(\frac{R_2}{R_1}\frac{k}{q}\frac{\ln\left(\frac{R_2}{R_1}\frac{A_{D2}}{A_{D1}}\right)}{R_0A_{D1}\tilde{J}_{SX}}\right)\right]$	$c_6 = \frac{k}{q}K(1-m)$
Doyle	$a_6 = V_{G0}$	$b_6 = \frac{k}{q}\left[\frac{K_3}{R_5}\frac{R_1R_3}{R_1+R_2+R_3}\ln\left(K_1\frac{A_{S2}}{A_{S1}}\right)+\frac{R_2+R_3}{R_1+R_2+R_3}\ln\left(\frac{K_4}{R_5}\frac{k}{q}\ln\left(K_1\frac{A_{S2}}{A_{S1}}\right)\right)-\ln(J_{SX}A_{S1})\right]$	$c_6 = \frac{k}{q}\left(\frac{R_2+R_3}{R_1+R_2+R_3}-m\right)$

$$V_{REF} = a + bT + cT\ lnT$$

- Start-up Circuits Required on all Bandgap References discussed here
- Bandgap circuits widely used to build voltage references for over 4 decades
- Basic bandgap circuits still used today
- Trimming often required to set inflection point at desired temperature
- Offset voltage of Op Amp and TCR of resistors degrade performance
- Experimental performance often a factor of 2 to 10 worse than that predicted here but still quite good
- Ongoing research activities focusing on improving performance of bandgap references

Stay Safe and Stay Healthy !

End of Lecture 41